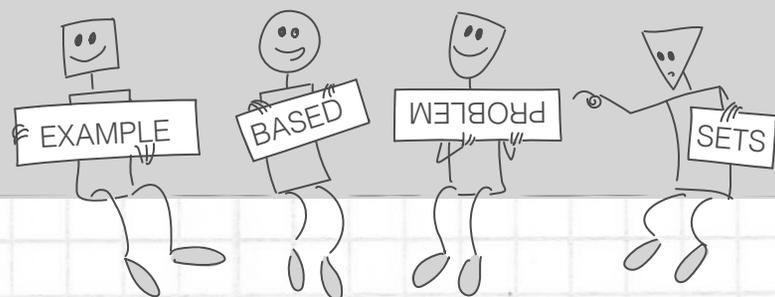


5th grade



MathBy Example

Teacher Edition

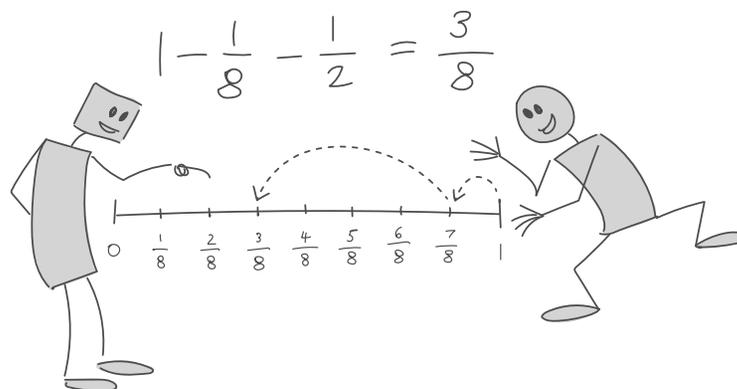
TEACHER EDITION

MathByExample

Example Based Problem Sets



5th Grade



Strategic Education Research Partnership
1100 Connecticut Avenue NW
Suite 1310
Washington, DC 20036
(202) 223-8555
info@serpinstitute.org

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Getting Started with MathByExample

Quick Facts

- MathByExample is specially designed to target common misconceptions and errors in mathematics through worked examples and prompts in which students are asked to explain a fictitious student's work.
- MathByExample is made up of 60+ assignments each for grades 4 and 5, with two sets of problems in each assignment.
- Each problem set targets common math mistakes through a worked example (marked as either correct or incorrect) and a practice problem.
- All assignments are aligned to the Common Core State Standards for Mathematics.
- The assignments went through numerous rounds of revisions based on feedback from teachers, coaches, and math experts.

Planning

- Review the assignments carefully to determine when each should be used during the school year, given the order of topics in your curriculum. A full assignment list with room for notes is provided.
- Assignments are grouped into "units" within the table of contents for organizational purposes, but individual assignments can be used in any order.
- You may use any other materials you would normally use and/or replace other practice worksheets with the MathByExample assignments.
- Assignments are generally designed to take approximately 15-20 minutes. But you may decide to use an assignment over two days or to use two assignments in one day. There may be weeks where none of the assignments align to what you're teaching, but other weeks when many of the assignments will fit well. MathByExample is designed to flexibly fit into your classroom without changes to the content you teach.
- You may decide to walk through the first assignment with students, making sure they understand that they will be looking at both correct (marked with a ✓) and incorrect (marked with an ✗) student work, as well as answering questions in writing and solving a similar problem on their own.

Using MathByExample

You may use individual assignments in various ways throughout the year. Some examples are:

- Warm-up / do now
- Exit ticket
- Homework
- During centers
- Formative assessment
- Review before a unit test (or standardized testing)

You may decide to have students work on the assignments:

- Independently
- With partners
- In groups

We also encourage you to review the assignments with your class after the students have finished to provide further opportunities for students to process and discuss the targeted concepts and common errors.

Frequently Asked Questions

When should I use the assignments?

You can use the assignments whenever makes the most sense for you. Although intended for math class, you can use the assignments at any time throughout the day, such as homeroom or during a workshop period.

How often?

There's no set frequency that we recommend — just whenever the assignments align to the content you're teaching. You may decide to use an assignment over two days or to use two assignments in one day. There may be weeks where none of the assignments align to what you're teaching, but other weeks when many of the assignments will fit well. On average, completing 2-3 assignments per week throughout the year would allow you to use all of the assignments with your students.

How long do the assignments take?

On average, assignments are designed to take about 15-20 minutes to complete.

Can I let my students work on the assignments together?

Absolutely. Many teachers have let us know that they find the assignments most beneficial when students work on the assignments together, since students start discussing the mathematics.

Do students have to answer the questions?

Research shows that at least attempting to answer the questions improves learning, so we suggest encouraging students to at least try to answer each question.

Do I have to use all of the assignments?

MathByExample was designed to be flexible — use as many or as few assignments as you like! However, past research has shown that the more students are exposed to worked examples the better! Also, note that you don't have to use them all when first teaching the content — you can use them at any time throughout the year, such as a refresher when reviewing for spring standardized tests.

What if I don't teach the content that some of the assignments cover?

We did our best to align the content of the assignments to the Common Core State Standards for each grade, but if you don't teach some of the content, don't feel obligated to use the assignments. We expect the scope and sequences to be different across each district and even within each district.

Does any research support this approach?

Yes! Preliminary analyses demonstrate statistically significant results for students using the MathByExample materials. We found that explaining correct and incorrect examples can be effective for students in upper elementary school. A previous study (AlgebraByExample) tested the effect of worked examples in Algebra assignments, too. Results of a randomized trial indicated significant impacts, the largest of which was for low achieving students. Learn more on the MathByExample website.

Any other questions?

Ask us! Email info@serpinstitute.org if you have any additional questions, comments, or recommendations!

About the MathByExample Teacher Edition

The "Target" summarizes the targeted mathematical concept. The "Targeted Error" notes a specific error, if applicable.

An image from each Student Workbook page is embedded within the Teacher Edition.

Multi-Digit Numbers
Multiplying Multi-Digit Numbers (Part 1), continued

Teacher Notes

SET 2 OF 2

TARGET:

Understanding partial products in the standard algorithm. (Targeted Error: Using single-digit multiplication rules when multiplying multi-digit numbers.)

1. Students study Olivia's work. Olivia has completed her work incorrectly.

2. Students respond to three questions related to Olivia's work:

- How could Olivia have known from the size of her answer that she was not correct?

Sample response:

22 is smaller than both 45 and 32. When you multiply whole numbers, your answer should be greater than both factors.

- Explain why Olivia cannot decompose 45 into 4 and 5.

Sample response:

45 is 40 and 5, not 4 and 5.

- Explain why Olivia cannot decompose 32 into 3 and 2.

Sample response:

32 is 30 and 2, not 3 and 2.

3. Students do a similar problem on their own.

Solution:

1,350

Multi-Digit Numbers
Multiplying Multi-Digit Numbers (Part 1), continued

SET TWO

1. Study Olivia's incorrect work.

X Name: Olivia

Solve.

$$45 \times 32 = 22$$

45
x 32

10
+ 12

22

2. Answer these questions.

- How could Olivia have known from the size of her answer that she was not correct?
- Explain why Olivia cannot decompose 45 into 4 and 5.
- Explain why Olivia cannot decompose 32 into 3 and 2.

3. Then complete this one.

Solve.

$$18 \times 75 = \underline{\hspace{2cm}}$$

Sample response(s) are actually samples! Students will respond in various ways, so it will be up to you to judge whether a student understands the concept.

Student directions often ask students to show a model or draw a diagram, but rarely do they specifically state to "show your work." We hope that students do show their work, but ultimately it is up to you to set expectations regarding showing their work and which model they use to solve a problem.

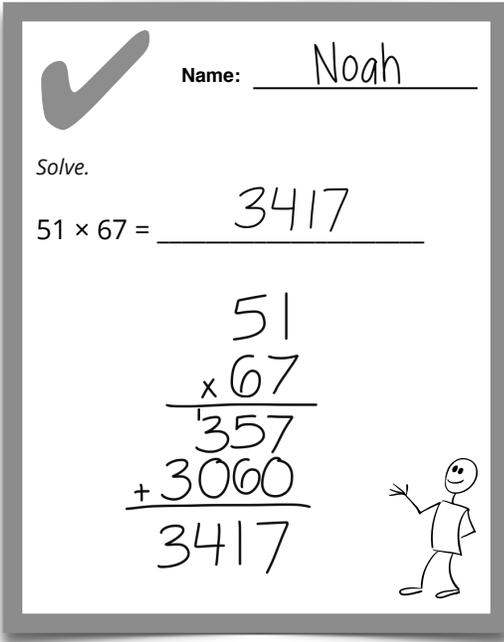
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Name: _____ Date: _____

SET ONE

1. Study Noah's correct work.



A whiteboard with a grey border. In the top left corner is a large grey checkmark. To its right, the name "Noah" is written in cursive on a line. Below the name, the word "Solve." is written. Underneath, the equation $51 \times 67 =$ is followed by a horizontal line with "3417" written above it. Below this, a standard multiplication algorithm is shown:
$$\begin{array}{r} 51 \\ \times 67 \\ \hline 357 \\ + 3060 \\ \hline 3417 \end{array}$$
 To the right of the algorithm is a simple stick figure with its arms raised.

2. Answer these questions.

1. What numbers did Noah multiply to get 357?

2. What numbers did Noah multiply to get 3,060?

3. Then complete this one.

Solve.

$39 \times 74 =$ _____

Multi-Digit Numbers

Multiplying Multi-Digit Numbers (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding partial products in the standard algorithm.

1. Students study Noah's work. Noah has completed his work correctly.

2. Students respond to two questions related to Noah's work:

1. What numbers did Noah multiply to get 357?

Sample response:

Noah multiplied 51 and 7.

2. What numbers did Noah multiply to get 3,060?

Sample response:

Noah multiplied 51 and 60.

3. Students do a similar problem on their own.

Solution:

2,886

Teacher Notes

SET 2 OF 2

TARGET:

Understanding partial products in the standard algorithm. (Targeted Error: Using single-digit multiplication rules when multiplying multi-digit numbers.)

1. Students study Olivia's work. Olivia has completed her work incorrectly.

2. Students respond to three questions related to Olivia's work:

- How could Olivia have known from the size of her answer that she was not correct?

Sample response:

22 is smaller than both 45 and 32. When you multiply whole numbers, your answer should be greater than both factors.

- Explain why Olivia cannot decompose 45 into 4 and 5.

Sample response:

45 is 40 and 5, not 4 and 5.

- Explain why Olivia cannot decompose 32 into 3 and 2.

Sample response:

32 is 30 and 2, not 3 and 2.

3. Students do a similar problem on their own.

Solution:

1,350

SET TWO

1. Study Olivia's incorrect work.

X

Name: Olivia

Solve.

$45 \times 32 = \underline{22}$

2. Answer these questions.

- How could Olivia have known from the size of her answer that she was not correct?
- Explain why Olivia cannot decompose 45 into 4 and 5.
- Explain why Olivia cannot decompose 32 into 3 and 2.

3. Then complete this one.

Solve.

$18 \times 75 = \underline{\hspace{2cm}}$

Name: _____ Date: _____

SET ONE

1. Study Liam's correct work.

A handwritten math problem by Liam. At the top left is a large checkmark. To its right, the name "Liam" is written on a line. Below the name, the word "Solve." is written. The main problem is $1,238 \times 12 = 14,856$. Below this, the multiplication is shown in a standard algorithm format. The multiplicand is 1238 and the multiplier is 12. The first partial product is 2476, and the second partial product is 12380. The final sum is 14856. A small grey arrow points to a "1" written above the digit 3 in the tens place of the multiplicand. To the right of the work is a simple stick figure drawing of a person.

2. Answer these questions.

Why did Liam write a 1 above the 3 in the step marked with an arrow?

How did Liam use that 1 in his calculations?

3. Then complete this one.

Solve.

$4,326 \times 14 =$ _____

Multi-Digit Numbers

Multiplying Multi-Digit Numbers (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to carry the value of 10 when using the standard algorithm to multiply.

1. Students study Liam's work. Liam has completed his work correctly.

2. Students respond to two questions related to Liam's work:

Why did Liam write a 1 above the 3 in the step marked with an arrow?

Sample response:

$8 \times 2 = 16$. Liam wrote 6 in the ones place and had to carry the 10.

How did Liam use that 1 in his calculations?

Sample response:

Liam added it to the 6 (3×2).

3. Students do a similar problem on their own.

Solution:

60,564

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to use the standard algorithm to multiply. (Targeted Error: Ignoring place value of digits in the tens and hundreds column for one factor when multiplying multi-digit numbers.)

1. Students study Emma's work. Emma has completed her work incorrectly.

2. Students respond to two questions related to Emma's work:

- Emma multiplied 4,268 by 1 in the step marked with an arrow. By what number should Emma have multiplied 4,268? Explain your reasoning.

Sample response:

10. The 1 is in the tens place.

- What number should Emma have written at the step marked with an arrow?

42,680

3. Students do a similar problem on their own.

Solution:

2,089,398

SET TWO

1. Study Emma's incorrect work.

X

Name: Emma

Solve.

$912 \times 4,268 = \underline{51,216}$

$$\begin{array}{r}
 \begin{array}{cccc}
 & 6 & 7 & \\
 4 & 2 & 6 & 8 \\
 \times & 9 & 1 & 2 \\
 \hline
 8 & 5 & 3 & 6 \\
 4 & 2 & 6 & 8 \\
 + & 3 & 8 & 4 & 1 & 2 \\
 \hline
 5 & 1 & 2 & 1 & 6
 \end{array}
 \end{array}$$

← 

2. Answer these questions.

- Emma multiplied 4,268 by 1 in the step marked with an arrow. By what number should Emma have multiplied 4,268? Explain your reasoning.

- What number should Emma have written at the step marked with an arrow?

3. Then complete this one.

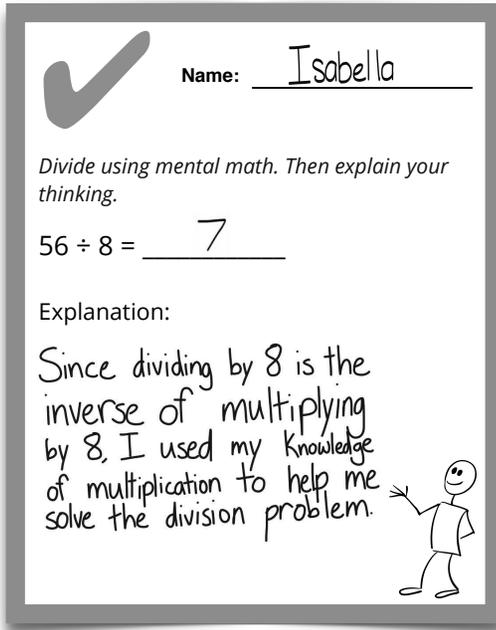
Solve.

$$727 \times 2,874 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

SET ONE

1. Study Isabella's correct work.



Name: Isabella

Divide using mental math. Then explain your thinking.

$56 \div 8 = \underline{7}$

Explanation:

Since dividing by 8 is the inverse of multiplying by 8, I used my knowledge of multiplication to help me solve the division problem.

2. Answer these questions.

- What does Isabella mean by “dividing by 8 is the **inverse** of multiplying by 8”?
- What multiplication fact did Isabella use to help her solve the division problem?

3. Then complete this one.

Divide using mental math. Then explain your thinking.

$63 \div 7 = \underline{\hspace{2cm}}$

Explanation:

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to find quotients using knowledge of multiplication.

1. Students study Isabella's work. Isabella has completed her work correctly.

2. Students respond to two questions related to Isabella's work:

- What does Isabella mean by “dividing by 8 is the **inverse** of multiplying by 8”?
Sample response:
Isabella means that dividing by 8 is the opposite of multiplying by 8.
- What multiplication fact did Isabella use to help her solve the division problem?
Sample responses:
She knew that 8×7 is 56.
She knew that 7×8 is 56.

3. Students do a similar problem on their own.

Solution:

9

Sample explanations:

Since dividing by 7 is the inverse of multiplying by 7, I used my knowledge of multiplication to help me solve the division problem.

Since dividing by 7 is the inverse of multiplying by 7, I used my knowledge that $7 \times 9 = 63$ to solve $63 \div 7 = 9$.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to use patterns of 10 when dividing. (Targeted Error: Shifting the wrong number of place values.)

1. Students study Ethan's work. Ethan has completed his work incorrectly.

2. Students respond to two questions related to Ethan's work:

- Ethan incorrectly shifted 1,200 two place values. How many place values should he have shifted? Explain how you know.

Sample response:

1. When dividing by 10, you only shift one place value to the right.

- What divisor would have required Ethan to shift two place values?

100

3. Students do a similar problem on their own.

Solution:

260

SET TWO

1. Study Ethan's incorrect work.

X Name: ETHAN

Solve.

$1,200 \div 10 = 12$

$1,200 \div 10$

$1,200 \div 10$

12

2. Answer these questions.

- Ethan incorrectly shifted 1,200 two place values. How many place values should he have shifted? Explain how you know.
- What divisor would have required Ethan to shift two place values?

3. Then complete this one.

Solve.

$$2,600 \div 10 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

SET ONE

1. Study William's correct work.

A handwritten student work sample for William. At the top left is a large checkmark. To its right, the name "William" is written in a cursive font. Below the name, the word "Solve." is written. The first problem is $792 \div 6 = \underline{\quad 132 \quad}$. Below this is a division problem with a box: $6 \overline{) 792}$ with a question mark above the box. Below the box is the equation $100 + 30 + 2 = 132$. Below that is another division problem with a box: $6 \overline{) 600 \mid 180 \mid 12}$. To the right of the box is a simple stick figure drawing of a person with arms and legs.

2. Answer these questions.

- 1. Why did William write 600, 180, and 12 inside the area model?
- 2. What do the numbers on the top of the area model represent?

3. Then complete this one.

Solve.

$676 \div 4 = \underline{\hspace{2cm}}$

Multi-Digit Numbers

Dividing Multi-Digit Numbers (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to use an area model to illustrate division.

1. Students study William's work. William has completed his work correctly.

2. Students respond to two questions related to William's work:

- 1. Why did William write 600, 180, and 12 inside the area model?

Sample responses:

All of those remainders together total the dividend.

They add up to 792.

- 2. What do the numbers on the top of the area model represent?

Sample responses:

They add up to equal the quotient.

The numbers on top are the numbers in the area model divided by 6.

3. Students do a similar problem on their own.

Solution:

169

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to divide using long division. (Targeted Error: Forgetting to note "0" in a multi-digit quotient.)

1. Students study Ava's work. Ava has completed her work incorrectly.

2. Students respond to a question related to Ava's work:

- Why did Ava write 0 in the step marked with an arrow?

Sample response:

She wrote 0 because 4 goes into 12 0 times.

- Ava forgot to record this zero in her quotient. Explain how this changed her answer.

Sample response:

Her answer was 29 instead of 209.

3. Students do a similar problem on their own.

Solution:

307

SET TWO

1. Study Ava's incorrect work.

X Name: Ava

Solve.

$$2,926 \div 14 = 29$$
$$\begin{array}{r} 29 \\ 14 \overline{) 2926} \\ \underline{-28} \\ 12 \\ \underline{-0} \\ 126 \\ \underline{-126} \\ 0 \end{array}$$

2. Answer the question.

- Why did Ava write 0 in the step marked with an arrow?
- Ava forgot to record this zero in her quotient. Explain how this changed her answer.

3. Then complete this one.

Solve.

$$4,912 \div 16 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

SET ONE

1. Study Simon's incorrect work.

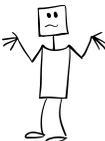
X Name: Simon

Solve. Write your answer in sentence form.

A college has a swimming pool that is 51 feet wide by 126 feet long. What is the area of the pool?



The area of the pool is 354 square feet.


$$\begin{array}{r} 126 \\ 126 \\ 51 \\ +51 \\ \hline 354 \end{array}$$

3. Then complete this one.

Solve. Write your answer in sentence form.

The high school's new soccer field is 65 yards wide by 115 yards long. What is the area of the field?

2. Answer these questions.

- Simon did not find the area of the pool. What measurement did Simon find instead? Explain your reasoning.
- What does area measure?
- How should Simon have found the area of the pool?

Multi-Digit Numbers

Multi-Digit Numbers Word Problems

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to identify the operation needed to solve a word problem. (Targeted Error: Misinterpreting the word problem and using the wrong operation.)

1. Students study Simon's work. Simon has completed his work incorrectly.

2. Students respond to three questions related to Simon's work:

- Simon did not find the area of the pool. What measurement did Simon find instead? Explain your reasoning.

Sample response:

Perimeter. Simon added the lengths of each side.

- What does area measure?

Sample response:

Area measures the size of the surface.

- How should Simon have found the area of the pool?

Sample responses:

$$(50 \times 126) + (1 \times 126)$$

$$51 \times 126$$

3. Students do a similar problem on their own.

Solution:

The area of the field is 7475 square yards.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to identify the operation needed to solve a word problem.

1. Students study Renata's work. Renata has completed her work correctly.

2. Students respond to two questions related to Renata's work:

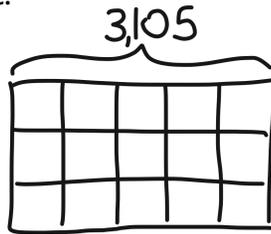
- Why did Renata divide in order to find the answer?

Sample response:

The total number of crayons must be split among the 15 classrooms.

- Draw a diagram to represent the word problem.

Sample response:



3. Students do a similar problem on their own.

Solution:

Each friend will get 104 candies.

SET TWO

1. Study Renata's correct work.

Name: Renata

Solve. Write your answer in sentence form.

A principal split 3,105 crayons evenly among 15 classrooms in the elementary school. How many crayons did each classroom receive?

$$\begin{array}{r}
 207 \\
 15 \overline{)3105} \\
 \underline{-30} \\
 10 \\
 \underline{-0} \\
 105 \\
 \underline{-105} \\
 0
 \end{array}$$

Each classroom received 207 crayons.

2. Answer these questions.

- Why did Renata divide in order to find the answer?
- Draw a diagram to represent the word problem.

3. Then complete this one.

Solve. Write your answer in sentence form.

A boy bought a large bag of chocolate candies to give out at his birthday party. The bag has 2,288 candies, and he is inviting 22 friends to his party. If split evenly, how many candies will each friend get?

Name: _____ Date: _____

SET ONE

1. Study Evelyn's correct work.

✓ Name: Evelyn

Use the place value chart to solve.

$4.36 \times 10 = \underline{43.6}$

		4	.	3	6	
4	3	.	6			

2. Answer the question.

- Why did Evelyn move the 6 from the hundredths place to the tenths place?

3. Then complete this one.

Use the place value chart to solve.

$3.42 \times 100 = \underline{\hspace{2cm}}$

			.			
			.			

Place Value
Multiplying by Powers of 10

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how the digits shift when you multiply by a power of 10.

- Students study Evelyn's work. Evelyn has completed her work correctly.**

- Students respond to a question related to Evelyn's work:**

- Why did Evelyn move the 6 from the hundredths place to the tenths place?

Sample response:

Evelyn multiplied by 10, so the value of 6 is now 10 times greater.

- Students do a similar problem on their own.**

Solution:

342

		3	.	4	2	
3	4	2	.			

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how the digits shift when you multiply by a power of 10.
 (Targeted Error: Misinterpreting the relationship between the power of 10 and the number of digits shifted.)

1. Students study Matthew's work. Matthew has completed his work incorrectly.

2. Students respond to a question related to Matthew's work:

- Matthew filled out the table correctly, but his answer is incorrect. How can you tell his answer is incorrect by looking at the table?

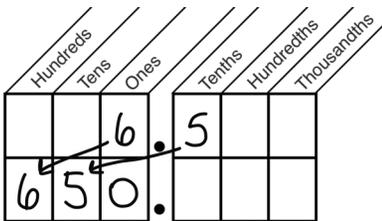
Sample response:

Each of his digits shifted 2 places to the left, so the value of each digit increased by 100.

3. Students do a similar problem on their own.

Solution:

100



SET TWO

1. Study Matthew's incorrect work.

X

Name: Matthew

Use the place value chart to solve.

$4.7 \times \underline{10} = 470$

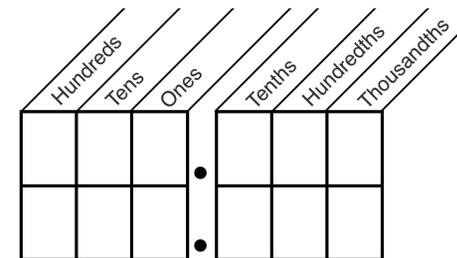
2. Answer the question.

- Matthew filled out the table correctly, but his answer is incorrect. How can you tell his answer is incorrect by looking at the table?

3. Then complete this one.

Use the place value chart to solve.

$$6.5 \times \underline{\quad\quad} = 650$$



Name: _____ Date: _____

SET ONE

1. Study Fatima's incorrect work.

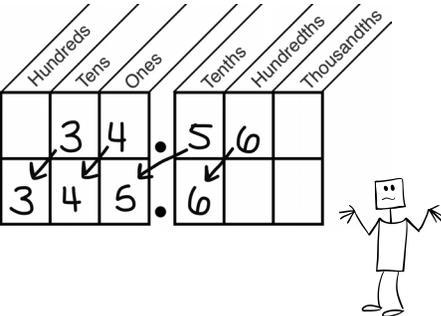
X

Name: Fatima

Use the place value chart to solve.

$34.56 \div 10 = \underline{345.6}$

	3	4	.	5	6	
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths



2. Answer these questions.

- 1. How can you tell that Fatima multiplied by 10 rather than divided by 10?
- 2. What should Fatima have done differently in the place value chart to show division?

Place Value
Dividing by Powers of 10

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how the digits shift when you divide by a power of 10. (Targeted Error: Shifting the digits in the wrong direction – multiplying instead of dividing.)

1. Students study Fatima's work. Fatima has completed her work incorrectly.

2. Students respond to two questions related to Fatima's work:

- 1. How can you tell that Fatima multiplied by 10 rather than divided by 10?

Sample responses:

She shifted the digits to the left.

The quotient is larger than the dividend.

- 2. What should Fatima have done differently in the place value chart to show division?

Sample response:

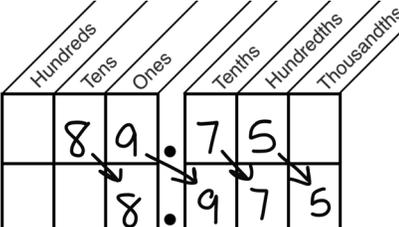
She should have shifted the digits to the right.

3. Students do a similar problem on their own.

Solution:

8.975

	8	9	.	7	5	
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths



3. Then complete this one.

Use the place value chart to solve.

$89.75 \div 10 = \underline{\hspace{2cm}}$

Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how the digits shift when you divide by a power of 10.

1. Students study Malik's work. Malik has completed his work correctly.

2. Students respond to a question related to Malik's work:

- How did Malik know to divide by 100, rather than by 10?

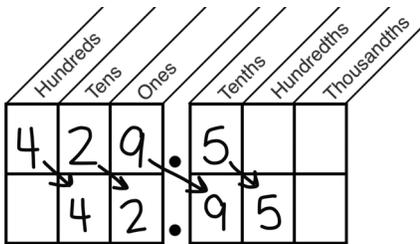
Sample response:

Malik knew because the digits shifted 2 place values to the right.

3. Students do a similar problem on their own.

Solution:

10



SET TWO

1. Study Malik's correct work.

Name: Malik

Use the place value chart to solve.

$$567.3 \div 100 = 5.673$$

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
5	6	7	3		
		5	6	7	3



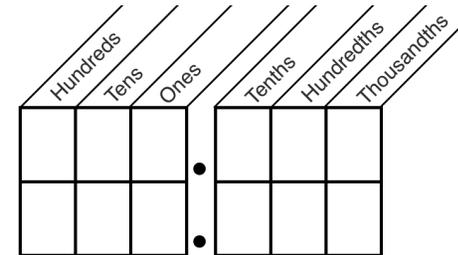
2. Answer the question.

- How did Malik know to divide by 100, rather than by 10?

3. Then complete this one.

Use the place value chart to solve.

$$429.5 \div \underline{\hspace{2cm}} = 42.95$$



Name: _____ Date: _____

Place Value
Understanding Exponents
CCSS 5.NBT.A.2

SET ONE

1. Study Victoria's incorrect work.

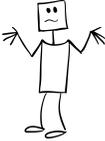
X Name: Victoria

Write the following in expanded and standard form.

10^4

Expanded form:
 10×4

Standard form:
40



2. Answer these questions.

- 1. Victoria mistakenly thought that $10^4 = 10 \times 4$. What does the 4 in 10^4 actually represent?
- 2. Write 10^4 in expanded form.
- 3. Write 10^4 in standard form.

3. Then complete this one.

Write the following in expanded and standard form.

10^5

Expanded form: _____

Standard form: _____

Place Value
Understanding Exponents

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that 10^x means 10 multiplied by itself x number of times.
(Targeted Error: Believing that 10^x means 10 multiplied by x .)

1. Students study Victoria's work. Victoria has completed her work incorrectly.

2. Students respond to three questions related to Victoria's work:

- 1. Victoria mistakenly thought that $10^4 = 10 \times 4$. What does the 4 in 10^4 actually represent?

Sample responses:

The 4 means you multiply the base 4 times.

The 4 represents the number of times 10 is multiplied by itself.

- 2. Write 10^4 in expanded form.

$10 \times 10 \times 10 \times 10$

- 3. Write 10^4 in standard form.

10,000

3. Students do a similar problem on their own.

Solution:

Expanded form: $10 \times 10 \times 10 \times 10 \times 10$

Standard form: 100,000

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the relationship between the number of zeros and the values of the exponents when working with the power of 10.

1. Students study David's work. David has completed his work correctly.

2. Students respond to two questions related to David's work:

- What pattern do you notice between the values of the exponents and the number of zeros in David's answers?

Sample response:

The number of zeros is equal to the value of the exponent.

- How many zeros would there be if you wrote 10^{25} in standard form?

25 zeros

3. Students do a similar problem on their own.

Solution:

$$10^5 = 100,000$$

$$10^6 = 1,000,000$$

$$10^7 = 10,000,000$$

SET TWO

1. Study David's correct work.

✓ Name: David

Write each of the following in standard form.

$$10^2 = \underline{100}$$
$$10^3 = \underline{1,000}$$
$$10^4 = \underline{10,000}$$

2. Answer these questions.

- What pattern do you notice between the values of the exponents and the number of zeros in David's answers?
- How many zeros would there be if you wrote 10^{25} in standard form?

3. Then complete this one.

Write each of the following in standard form.

$$10^5 = \underline{\hspace{2cm}}$$

$$10^6 = \underline{\hspace{2cm}}$$

$$10^7 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

Multiplying and Dividing by Powers of 10 (Exponents)

CCSS 5.NBT.A.2

SET ONE

1. Study Chloe's correct work.



Name: Chloe

Use the place value chart to solve.

$6.3 \div 10^2 = \underline{0.063}$

	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			6	.	3	
		0	.	0	6	3



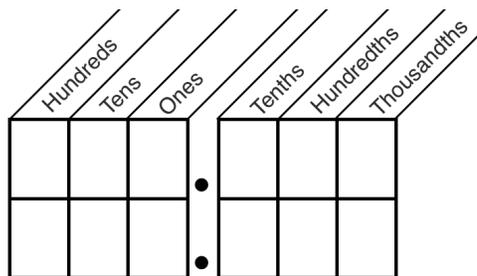
2. Answer the question.

- 1. How did Chloe know to shift the digits two places?

3. Then complete this one.

Use the place value chart to solve.

$74 \div 10^3 = \underline{\hspace{2cm}}$



Place Value

Multiplying and Dividing by Powers of 10 (Exponents)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding the relationship between the number of digits to shift and the exponent when working with the power of 10.

1. Students study Chloe's work. Chloe has completed her work correctly.

2. Students respond to a question related to Chloe's work:

- 1. How did Chloe know to shift the digits two places?

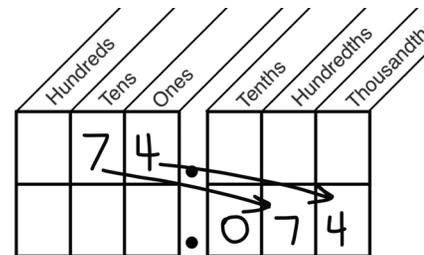
Sample response:

The exponent tells you how many places to shift the digits.

3. Students do a similar problem on their own.

Solution:

0.074



Teacher Notes

SET 2 OF 2

TARGET:

Understanding the relationship between the number of digits to shift and the exponent when working with the power of 10. (Targeted Error: Adding zeros after a decimal number when multiplying by a power of 10, without shifting the digits.)

1. Students study Diego's work. Diego has completed his work incorrectly.

2. Students respond to two questions related to Diego's work:

- Diego thought he was just supposed to add 2 zeros to the right side of the 7. Is the value of 6.7 different from the value of 6.700? Explain.

Sample response:

No, they have the same value. The decimal is in the same place. The zeros Diego added are just place holders and do not change the value of the number.

- What should Diego have done to multiply by 10^2 instead of just adding two zeros?

Sample response:

He should have shifted the digits two places to the left.

3. Students do a similar problem on their own.

Solution:

2,400

SET TWO

1. Study Diego's incorrect work.

X

Name: Diego

Write the following in standard form.

$6.7 \times 10^2 =$ 6.700

6.7×10^2

6.7×100

6.700



2. Answer these questions.

- Diego thought he was just supposed to add 2 zeros to the right side of the 7. Is the value of 6.7 different from the value of 6.700? Explain.
- What should Diego have done to multiply by 10^2 instead of just adding two zeros?

3. Then complete this one.

Write the following in standard form.

$2.4 \times 10^3 =$ _____

Name: _____ Date: _____

SET ONE

1. Study Grace's incorrect work.

X

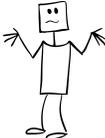
Name: Grace

Write the following in word form.

Standard Form: 37.2

Word Form:

three hundred seventy two



2. Answer these questions.

- 1. What mistake did Grace make when writing the number in word form?
- 2. What is the standard form of three hundred seventy two?
- 3. What is the correct way to write 37.2 in word form?

3. Then complete this one.

Write the following in word form.

Standard Form: 98.3

Word Form: _____

Decimals

Writing Decimals

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to convert a number in standard form to word form. (Targeted Error: Ignoring the decimal point when stating the word form of a number.)

1. Students study Grace's work. Grace has completed her work incorrectly.

2. Students respond to three questions related to Grace's work:

- 1. What mistake did Grace make when writing the number in word form?

Sample responses:

*She treated it like a whole number and ignored the decimal.
She did not take into account the place value of each number.*

- 2. What is the standard form of three hundred seventy two?

372

- 3. What is the correct way to write 37.2 in word form?

thirty seven and two tenths

3. Students do a similar problem on their own.

Solution:

ninety eight and three tenths

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the link between place value and decimal form.

1. Students study Thao's work. Thao has completed her work correctly.

2. Students respond to a question related to Thao's work:

🗣️ How did Thao know to write .2 as (2×0.1) ?

Sample response:

The 2 is in the tenths place, which is the same as .2 or 2×0.1 .

3. Students do a similar problem on their own.

Solution:

$$(4 \times 1) + (6 \times 0.1) + (5 \times 0.01) + (7 \times 0.001)$$

SET TWO

1. Study Thao's correct work.



Name: Thao

Write the following in expanded form.

Standard Form: 5.231

Expanded Form: $(5 \times 1) + (2 \times 0.1) + (3 \times 0.01) + (1 \times 0.001)$



2. Answer the question.

🗣️ How did Thao know to write .2 as (2×0.1) ?

3. Then complete this one.

Write the following in expanded form.

Standard Form: 4.657

Expanded Form: _____

Name: _____ Date: _____

SET ONE

1. Study Joseph's incorrect work.

X

Name: Joseph

Compare the numbers by writing $<$, $>$, or $=$ in the circle.

0.594 > 0.98



2. Answer these questions.

- 1. Joseph thought that numbers with more digits are always bigger. Why is this not always true?

- 2. Why is 0.98 actually larger than 0.594?

3. Then complete this one.

Compare the numbers by writing $<$, $>$, or $=$ in the circle.

0.45 0.321

Decimals
Comparing Decimals

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to compare decimals in standard form. (Targeted Error: Comparing the 'length' or number of digits in the decimal rather than using knowledge of place value to compare values.)

1. Students study Joseph's work. Joseph has completed his work incorrectly.

2. Students respond to two questions related to Joseph's work:

- 1. Joseph thought that numbers with more digits are always bigger. Why is this not always true?

Sample responses:

This is not always true because, when dealing with decimal numbers, the more digits there are to the right of the decimal, the more precise the number. It does not mean it's larger.

This is not always true because you need to first compare the largest place value and then compare smaller place values to the right, if needed.

- 2. Why is 0.98 actually larger than 0.594?

Sample responses:

0.98 = 980 thousandths, and 0.598 = 598 thousandths. 980 thousandths is greater than 598 thousandths.

0.594 has a 5 in the tenths place, and 0.98 has a 9 in the tenths place. 9 tenths is greater than 5 tenths.

3. Students do a similar problem on their own.

Solution:

>

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to compare decimals in standard form. (Targeted Error: Identifying a decimal with zeros placed to the right of the decimal number as having a different value than decimals without the zeros.)

1. Students study Antoinette's work. Antoinette has completed her work correctly.

2. Students respond to a question related to Antoinette's work:

- How did Antoinette know that 0.700 and 0.7 are equivalent even though 0.700 has more zeros?

Sample responses:

She knew because the zeros to the right of the 7 are just place holders. They do not change the value of the number.

$.700 = 700/1000$, and $.7 = 7/10$, and $700/1000$ is equivalent to $7/10$.

3. Students do a similar problem on their own.

Solution:

=

SET TWO

1. Study Antoinette's correct work.

Antoinette's work is shown in a rectangular frame. In the top left corner, there is a large grey checkmark. To its right, the text "Name: Antoinette" is written in a cursive font. Below the name, the instruction "Compare the numbers by writing <, >, or = in the circle." is written. In the center, the numbers "0.700" and "0.7" are separated by a circle containing an equals sign. In the bottom right corner, there is a simple stick figure drawing of a person with arms raised.

2. Answer the question.

- How did Antoinette know that 0.700 and 0.7 are equivalent even though 0.700 has more zeros?

3. Then complete this one.

Compare the numbers by writing <, >, or = in the circle.

$$0.030 \bigcirc 0.03$$

Name: _____ Date: _____

SET ONE

1. Study Zoey's correct work.

Name: Zoey

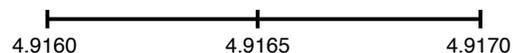
Round to the **nearest thousandth**. Then explain your reasoning.

$4.9162 \approx \underline{4.916}$

Explanation:
 $4.91\overline{6}2$
6 is in the thousandths place.
I rounded down to 4.916 because
2 is less than 5.

2. Answer these questions.

- Mark 4.9162 on the number line.



- How does using the number line help you determine that 4.916 is the nearest thousandth, rather than 4.917?

3. Then complete this one.

Round to the **nearest thousandth**. Then explain your reasoning.

$7.9243 \approx \underline{\hspace{2cm}}$

Explanation:

Decimals

Rounding Decimals (Part 1)

Teacher Notes

SET 1 OF 2

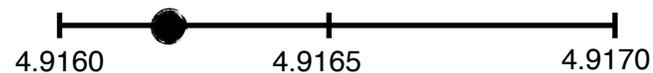
TARGET:

Understanding how to use the number line to aid in rounding.

1. Students study Zoey's work. Zoey has completed her work correctly.

2. Students respond to two questions related to Zoey's work:

- Mark 4.9162 on the number line.



- How does using the number line help you determine that 4.916 is the nearest thousandth, rather than 4.917?

Sample response:

It helps you see that 4.9162 is closer to 4.916 than 4.917.

3. Students do a similar problem on their own.

Solution:

7.924

Sample explanation:

4 is in the thousandths place. I rounded to 7.924 because 3 is less than 5.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to use the number line to aid in rounding. (Targeted Error: Changing the value of the digit in the designated place when rounding down.)

1. Students study Samuel's work. Samuel has completed his work incorrectly.

2. Students respond to two questions related to Samuel's work:

1. Mark 5.614 on the number line.



2. Based on the number line, explain why Samuel's answer should be 5.6 rather than 5.5.

Sample response:

5.614 is between 5.6 and 5.7. The nearest tenth is 5.6.

3. Students do a similar problem on their own.

Solution:

6.7

Sample explanation:

7 is in the tenths place. 1 rounded down because 4 is less than 5.

SET TWO

1. Study Samuel's incorrect work.

X

Name: Samuel

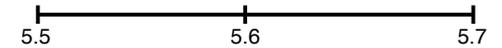
Round to the **nearest tenth**. Then explain your reasoning.

5.614 \approx 5.5

Explanation: 5.614
6 is in the tenths place.
1 is less than 5, so I rounded the 6 down to 5.

2. Answer these questions.

1. Mark 5.614 on the number line.



2. Based on the number line, explain why Samuel's answer should be 5.6 rather than 5.5.

3. Then complete this one.

Round to the **nearest tenth**. Then explain your reasoning.

6.741 \approx _____

Explanation:

Name: _____ Date: _____

SET ONE

1. Study Andrew's correct work.

✓ Name: Andrew

Round to the **nearest hundredth**. Then explain your reasoning.

$27.938 \approx \underline{27.94}$

Explanation:
27.938
3 is in the hundredths place. I rounded up to 27.94 because 8 is greater than 5.

2. Answer the question.

- Why didn't Andrew include the 8 in his answer?

3. Then complete this one.

Round to the **nearest hundredth**. Then explain your reasoning.

$93.106 \approx \underline{\hspace{2cm}}$

Explanation:

Decimals

Rounding Decimals (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to round decimal numbers. (Targeted Error: When rounding decimal numbers, including the values to the right of the digit in the designated place.)

1. Students study Andrew's work. Andrew has completed his work correctly.

2. Students respond to a question related to Andrew's work:

- Why didn't Andrew include the 8 in his answer?

Sample responses:

When rounding to the nearest hundredth, you do not include the thousandths digit.

Since he was rounding to the nearest hundredth, that means that all digits after the hundredths should be removed.

3. Students do a similar problem on their own.

Solution:

93.11

Sample explanation:

0 is in the hundredths place. I rounded up to 93.11 because 6 is greater than 5.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to round decimal numbers. (Targeted Error: Rounding to the tens place, instead of to the nearest tenth.)

1. Students study Natalie's work. Natalie has completed her work incorrectly.

2. Students respond to two questions related to Natalie's work:

- 1. Natalie was supposed to round to the nearest tenth. What place value did she round to instead?

tens

- 2. Explain how Natalie should have rounded to the nearest tenth.

Sample response:

She should have known that 3 was in the tenths place. Since 8 is larger than 5, she should have rounded up to 459.4.

3. Students do a similar problem on their own.

Solution:

327.2

Sample explanation:

1 is in the tenths place. 1 rounded up because 6 is greater than 5.

SET TWO

1. Study Natalie's incorrect work.

X Name: Natalie

Round to the **nearest tenth**. Then explain your reasoning.

459.38 ≈ 460

Explanation:

459.38
I rounded up because 9 is greater than 5.

2. Answer these questions.

- 1. Natalie was supposed to round to the nearest tenth. What place value did she round to instead?
- 2. Explain how Natalie should have rounded to the nearest tenth.

3. Then complete this one.

Round to the **nearest tenth**. Then explain your reasoning.

327.16 ≈ _____

Explanation:

Name: _____ Date: _____

SET ONE

1. Study Brooklyn's incorrect work.

X Name: Brooklyn

Solve.

$$15 + 0.1 = \underline{16}$$
$$\begin{array}{r} 15 \\ + 1 \\ \hline 16 \end{array}$$

2. Answer these questions.

1. What mistake did Brooklyn make when solving this problem?

2. Why do you have to pay attention to the decimal point?

3. Then complete this one.

Solve.

$$13 + 0.23 = \underline{\hspace{2cm}}$$

Decimals

Adding Decimals (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

*Understanding how to add whole numbers and decimal numbers.
(Targeted Error: Adding without considering place value.)*

1. Students study Brooklyn's work. Brooklyn has completed her work incorrectly.

2. Students respond to two questions related to Brooklyn's work:

1. What mistake did Brooklyn make when solving this problem?

Sample responses:

She ignored the decimal point (place value) and just added 15 and 1.

She didn't line up the decimal points.

2. Why do you have to pay attention to the decimal point?

Sample response:

The value of each digit changes depending on the place value.

3. Students do a similar problem on their own.

Solution:

$$13.23$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to apply the commutative property to simplify the work of addition.

1. Students study Gabriel's work. Gabriel has completed his work correctly.

2. Students respond to two questions related to Gabriel's work:

- Gabriel decided to reorder the numbers before adding. Why do you think he did this?

Sample responses:

He saw that he could make 10 with 4.5 and 5.5.

It was easier to add 4.5 and 5.5 first.

- Why is he allowed to reorder the numbers?

Sample responses:

He is allowed because addition is commutative. The order does not matter.

The order does not matter. $2 + 3$ is the same thing as $3 + 2$.

3. Students do a similar problem on their own.

Solution:

14.5

SET TWO

1. Study Gabriel's correct work.

✓ Name: GABRIEL

Solve.

$$4.5 + 3.7 + 5.5 = \underline{13.7}$$
$$\begin{array}{r} 4.5 + 5.5 + 3.7 \\ \swarrow \quad \searrow \\ 10 + 3.7 \\ \swarrow \quad \searrow \\ 13.7 \end{array}$$


2. Answer these questions.

- Gabriel decided to reorder the numbers before adding. Why do you think he did this?

- Why is he allowed to reorder the numbers?

3. Then complete this one.

Solve.

$$1.2 + 4.5 + 8.8 = \underline{\hspace{2cm}}$$

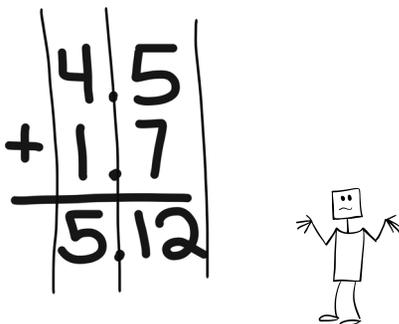
Name: _____ Date: _____

SET ONE

1. Study Joshua's incorrect work.

X Name: Joshua

Solve.

$$4.5 + 1.7 = \underline{5.12}$$


2. Answer these questions.

Based on the value of the numbers, how could Joshua have realized that $0.5 + 0.7$ does not equal 0.12 ?

What number should he have put under $0.5 + 0.7$? Explain how you know.

Decimals

Adding Decimals (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to regroup when adding decimals. (Targeted Error: Adding without regrouping.)

1. Students study Joshua's work. Joshua has completed his work incorrectly.

2. Students respond to two questions related to Joshua's work:

Based on the value of the numbers, how could Joshua have realized that $0.5 + 0.7$ does not equal 0.12 ?

Sample response:

0.12 is smaller than 0.5 and 0.7.

What number should he have put under $0.5 + 0.7$? Explain how you know.

Sample response:

He should've written 0.2 because 5 tenths plus 7 tenths is twelve tenths, which is the same thing as 1 ones and 2 tenths.

3. Students do a similar problem on their own.

Solution:

6.3

3. Then complete this one.

Solve.

$$3.7 + 2.6 = \underline{\hspace{2cm}}$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the properties of place value (e.g., 10 hundredths = 1 tenth).

1. Students study Lillian's work. Lillian has completed her work correctly.

2. Students respond to two questions related to Lillian's work:

- 10 hundredths equal how many tenths?

1

- In the step marked with an arrow, how did Lillian know there were 3 tenths?

Sample response:

Lillian knew because 10 hundredths = 1 tenth, so 30 hundredths = 3 tenths.

3. Students do a similar problem on their own.

Solution:

0.41

SET TWO

1. Study Lillian's correct work.

✓ Name: Lillian

Solve.

$$0.25 + 0.07 = 0.32$$
$$\begin{array}{r} 25 \text{ hundredths} \\ + 7 \text{ hundredths} \\ \hline 32 \text{ hundredths} \end{array}$$

32 hundredths
↓
3 tenths 2 hundredths
↓
0.32

2. Answer these questions.

- 10 hundredths equal how many tenths?

- In the step marked with an arrow, how did Lillian know there were 3 tenths?

3. Then complete this one.

Solve.

$$0.32 + 0.09 = \underline{\hspace{2cm}}$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to subtract decimals. (Targeted Error: Subtracting decimals without considering place value.)

1. Students study Carter's work. Carter has completed his work incorrectly.

2. Students respond to three questions related to Carter's work:

- Write 0.3 in word form.

three tenths

- Write 0.01 in word form.

one hundredth

- How would writing out the numbers in word form help Carter know he cannot just subtract 1 from 3?

Sample responses:

Carter cannot directly subtract units that are not the same.

In this problem, 3 and 1 are not in the same place - tenths are bigger than hundredths.

3. Students do a similar problem on their own.

Solution:

0.37

SET TWO

1. Study Carter's incorrect work.

X Name: Carter

Solve.

$0.3 - 0.01 = 0.2$

$$\begin{array}{r} 3 \\ - 1 \\ \hline 2 \end{array}$$

2. Answer these questions.

- Write 0.3 in word form.

- Write 0.01 in word form.

- How would writing out the numbers in word form help Carter know he cannot just subtract 1 from 3?

3. Then complete this one.

Solve.

$0.4 - 0.03 = \underline{\hspace{2cm}}$

Name: _____ Date: _____

SET ONE

1. Study John's correct work.

✓ Name: John

Solve.

$$7.2 - 6.4 = 0.8$$
$$\begin{array}{r} 6 \ 12 \\ \cancel{7}.\cancel{2} \\ -6.4 \\ \hline 0.8 \end{array}$$

2. Answer these questions.

1. In the step marked with an arrow, why did John cross off the 2 and write 12?

2. Why did John cross off the 7 and write 6?

3. Then complete this one.

Solve.

$$9.3 - 6.7 = \underline{\hspace{2cm}}$$

Decimals
Subtracting Decimals (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to regroup when subtracting decimals.

1. Students study John's work. John has completed his work correctly.

2. Students respond to two questions related to John's work:

1. In the step marked with an arrow, why did John cross off the 2 and write 12?

Sample response:

You cannot subtract 4 tenths from 2 tenths, so you have to regroup by moving 1 from the 7 and make it 12 tenths.

2. Why did John cross off the 7 and write 6?

Sample response:

He regrouped one whole from 7 to make 12 tenths, so he is left with 6 wholes.

3. Students do a similar problem on their own.

Solution:

2.6

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to regroup when subtracting decimals. (Targeted Error: Subtracting the smaller value from the larger value regardless of order.)

1. Students study Natalia's work. Natalia has completed her work incorrectly.

2. Students respond to two questions related to Natalia's work:

- Why can't Natalia subtract 0.5 from 0.9 in order to solve this problem?

Sample response:

Subtraction is not commutative. You cannot switch the order of numbers.

- Explain in words what Natalia should have done instead.

Sample response:

Natalia should have regrouped the 6 and made the tenths digit 15 and the ones digit 5.

3. Students do a similar problem on their own.

Solution:

2.5

SET TWO

1. Study Natalia's incorrect work.

X Name: Natalia

Solve.

$$6.5 - 3.9 = 3.4$$
$$\begin{array}{r} 6.5 \\ -3.9 \\ \hline 3.4 \end{array}$$

2. Answer these questions.

- Why can't Natalia subtract 0.5 from 0.9 in order to solve this problem?

- Explain in words what Natalia should have done instead.

3. Then complete this one.

Solve.

$$7.3 - 4.8 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

SET ONE

1. Study Tiana's correct work.

 Name: Tiana

Change the written expression into a numerical expression.

sum of three tenths and four hundredths

$0.3 + 0.04$



2. Answer the question.

- What phrase in the written expression indicated to Tiana that she should add?

3. Then complete this one.

Change the written expression into a numerical expression.

difference between seven tenths and nine hundredths

Teacher Notes

SET 1 OF 2

TARGET:

Determining which operation to use based on the written expression.
(Targeted Error: Using the wrong operation.)

1. Students study Tiana's work. Tiana has completed her work correctly.

2. Students respond to a question related to Tiana's work:

- What phrase in the written expression indicated to Tiana that she should add?

sum of

3. Students do a similar problem on their own.

Solution:

$0.7 - 0.09$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to convert a written expression to a numerical expression. (Targeted Error: Misplacing parentheses.)

1. Students study Tom's work. Tom has completed his work incorrectly.

2. Students respond to two questions related to Tom's work:

- Tom should not have used parentheses in his expression. Based on the written expression, how can you tell?

Sample response:

The expression says you should multiply first then add.

Parentheses force you to add first.

- What would the written expression have to say for $0.1 \times (0.04 + 0.2)$?

one tenth times the sum of four hundredths and two tenths

3. Students do a similar problem on their own.

Solution:

$$0.03 + 0.5 \times 0.6$$

SET TWO

1. Study Tom's incorrect work.

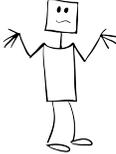
X

Name: Tom

Change the written expression into a numerical expression.

one tenth times four hundredths plus
two tenths

$0.1 \times (0.04 + 0.2)$



2. Answer these questions.

- Tom should not have used parentheses in his expression. Based on the written expression, how can you tell?

- What would the written expression have to say for $0.1 \times (0.04 + 0.2)$?

3. Then complete this one.

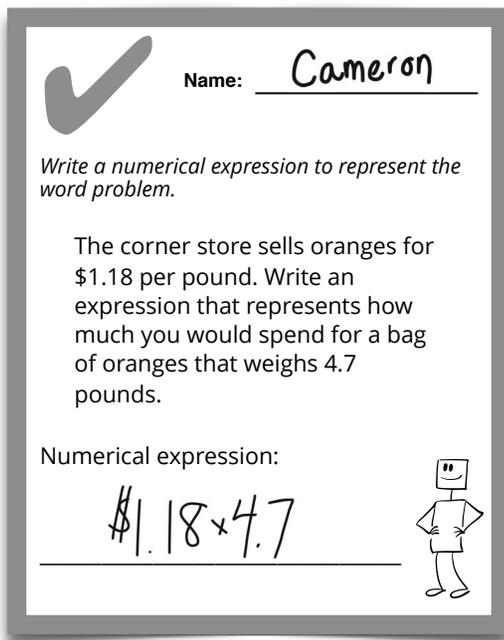
Change the written expression into a numerical expression.

three hundredths plus five tenths times six tenths

Name: _____ Date: _____

SET ONE

1. Study Cameron's correct work.



 Name: Cameron

Write a numerical expression to represent the word problem.

The corner store sells oranges for \$1.18 per pound. Write an expression that represents how much you would spend for a bag of oranges that weighs 4.7 pounds.

Numerical expression:

\$1.18 × 4.7



2. Answer the question.

- Could Cameron have also written $4.7 \times \$1.18$? Explain.

3. Then complete this one.

Write a numerical expression to represent the word problem.

Amazon is selling songs for \$0.55 each. Write an expression that represents how much you would spend for 15 songs.

Numerical expression: _____

Teacher Notes

SET 1 OF 2

TARGET:

Understanding the commutative property of multiplication.

1. Students study Cameron's work. Cameron has completed his work correctly.

2. Students respond to a question related to Cameron's work:

- Could Cameron have also written $4.7 \times \$1.18$? Explain.

Sample responses:

Yes, multiplication is commutative. The order does not matter.

Yes, $3 \times 4 = 4 \times 3$, so he could switch the order of the numbers.

3. Students do a similar problem on their own.

Solution:

$\$0.55 \times 15$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to write a numerical expression based on a word problem. (Targeted Error: Applying the concept of "write the numbers you hear" when writing numbers in standard form.)

1. Students study Kennedy's work. Kennedy has completed her work incorrectly.

2. Students respond to three questions related to Kennedy's work:

- 🗣️ Kennedy thought she just had to add zeros at the end of each number. What is the word form of the expression Kennedy wrote?

ninety seven plus five

- 🗣️ How is the word form for Kennedy's numerical expression different from what it says in the word problem?

Sample response:

The word problem uses hundredths and tenths while Kennedy's answer uses whole numbers.

- 🗣️ What numerical expression should Kennedy have written?

$0.97 + 0.5$ or $0.5 + 0.97$

3. Students do a similar problem on their own.

Solution:

$0.6 + 0.45$ or $0.45 + 0.6$

SET TWO

1. Study Kennedy's incorrect work.

X

Name: Kennedy

Write a numerical expression to represent the word problem.

A tennis player runs daily as part of an exercise plan. On Sunday she ran 97 hundredths of a mile. On Monday she ran 5 tenths of a mile. Write an expression that represents how much she ran in total.

Numerical expression:

97.00 + 5.0



2. Answer these questions.

- 🗣️ Kennedy thought she just had to add zeros at the end of each number. What is the word form of the expression Kennedy wrote?
- 🗣️ How is the word form for Kennedy's numerical expression different from what it says in the word problem?
- 🗣️ What numerical expression should Kennedy have written?

3. Then complete this one.

Write a numerical expression to represent the word problem.

A student has to walk 6 tenths of a mile from home in order to get to school and another 45 hundredths of a mile further in order to get to the playground. Write an expression that represents how far he has to walk from home to the playground.

Numerical expression: _____

SET ONE**1. Study Alexander's incorrect work.**

X

Name: Alexander

Solve and explain your reasoning.

$3.14 \times 10 = \underline{30.140}$

$3.14 \times 10 = 30.140$

Explanation:

I multiplied both sides of the decimal point by 10, and then put those numbers on either side of the decimal point.



3. Then complete this one.*Solve and explain your reasoning.*

$6.28 \times 10 = \underline{\hspace{2cm}}$

Explanation:

2. Answer these questions.

1. $3 \times 10 = 30$, so why isn't $0.14 \times 10 = 0.140$?

2. What is 0.14×10 ?

3. What should Alexander's answer have been?

Decimals

Multiplying Decimals

Teacher Notes**SET 1 OF 2****TARGET:**

*Understanding the role of place value when multiplying decimals.
(Targeted Error: Multiplying both sides of the decimal point individually by the other factor.)*

1. Students study Alexander's work. Alexander has completed his work incorrectly.

2. Students respond to three questions related to Alexander's work:

1. $3 \times 10 = 30$, so why isn't $0.14 \times 10 = 0.140$?

Sample response:

When you multiply by 10, it has to change the value. But adding a zero to the right of a decimal number doesn't change the value of the number.

2. What is 0.14×10 ?

1.4

3. What should Alexander's answer have been?

31.4

3. Students do a similar problem on their own.

Solution:

62.8

Sample explanations:

6 ones $\times 10 = 60$ ones. 28 hundredths $\times 10 = 280$ hundredths. 60 ones plus 280 hundredths = 62.8.

I multiplied 6.28 by 10, which shifted the digits one place to the left.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the role of place value when multiplying decimals.
(Targeted Error: Not knowing where to put the decimal point in the product.)

1. Students study Bianca's work. Bianca has completed her work correctly.

2. Students respond to a question related to Bianca's work:

- 🗣️ In the step marked with an arrow, how did Bianca know to place the decimal point between the 4 and 3?

Sample responses:

The unit value is hundredths.

In the factor 0.16, there are two digits to the right of the decimal point, so the product also needs two digits to the right of the decimal point.

3. Students do a similar problem on their own.

Solution:

10.20

SET TWO

1. Study Bianca's correct work.

✓ Name: Bianca

Solve.

$$27 \times 0.16 = \underline{4.32}$$
$$\begin{array}{r} 4 \\ .16 \\ \times 27 \\ \hline 112 \\ + 320 \\ \hline 4.32 \end{array}$$

→



2. Answer the question.

- 🗣️ In the step marked with an arrow, how did Bianca know to place the decimal point between the 4 and 3?

3. Then complete this one.

Solve.

$$68 \times 0.15 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

SET ONE

1. Study Emily's incorrect work.

X

Name: Emily

Solve. Draw an area model to explain your thinking.

$12 \times 0.17 = \underline{0.96}$

10	1 hundredth + 7 hundredths	
+	10	70
2	2	14

80 hundredths

16 hundredths

→ 96 hundredths

= 0.96

2. Answer the question.

- 🗣️ Emily did not correctly decompose 0.17 at the top of her area model. Explain why she should have used 10 hundredths + 7 hundredths.

3. Then complete this one.

Solve. Draw an area model to explain your thinking.

$17 \times 0.14 = \underline{\hspace{2cm}}$

Decimals

Modeling Multiplying Decimals

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to multiply whole numbers and decimals using an area model. (Targeted Error: Decomposing a decimal incorrectly.)

- 1. Students study Emily's work. Emily has completed her work incorrectly.**

2. Students respond to a question related to Emily's work:

- 🗣️ Emily did not correctly decompose 0.17 at the top of her area model. Explain why she should have used 10 hundredths + 7 hundredths.

Sample response:

The 1 is in the tenths place, so .1 is equal to 10 hundredths, not 1 hundredth.

3. Students do a similar problem on their own.

Solution:

2.38

7	10 + 4 hundredths	
+	70	28
10	100	40

98 hundredths

140 hundredths

> 238 hundredths

= 2.38

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the relationship between multiplication and repetitive addition.

1. Students study Michael's work. Michael has completed his work correctly.
2. Students respond to two questions related to Michael's work:

- How did Michael know to shade 0.25 **nine** times?

Sample responses:

His multiplication factor was 9.

He was multiplying 0.25 by 9.

- Michael added the shaded squares on each grid to calculate his product. Using repetitive addition, what number could he have added nine times to get the same product?

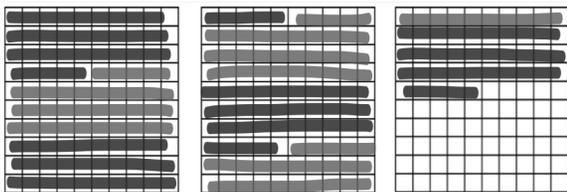
Sample response:

He could have added 0.25 nine times.

3. Students do a similar problem on their own.

Solution:

The pencils will cost a total of \$2.45.



SET TWO

1. Study Michael's correct work.

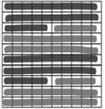


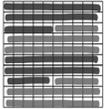
Name: Michael

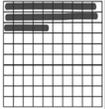
Use the grid diagram to solve the problem. Each grid is 10 × 10 and represents 1 whole. Write your answer in sentence form.

Each lap around the track is 0.25 miles. If a girl runs 9 laps, how far has she run?

0.25×9


1.0


1.0


0.25

$$\begin{array}{r} 10 \\ 10 \\ +0.25 \\ \hline 2.25 \end{array}$$

She ran 2.25 miles.



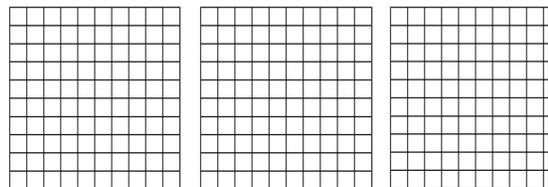
2. Answer these questions.

- How did Michael know to shade 0.25 **nine** times?
- Michael added the shaded squares on each grid to calculate his product. Using repetitive addition, what number could he have added nine times to get the same product?

3. Then complete this one.

Use the grid diagram to solve the problem. Each grid is 10 × 10 and represents 1 whole. Write your answer in sentence form.

A boy needs to buy a pencil for each student in his class. Each pencil costs \$0.35, and there are 7 children in his class. How much will all of the pencils cost?



Name: _____ Date: _____

SET ONE

1. Study Xavier's correct work.

✓ Name: Xavier

Solve.

$1.6 \div 2 = \underline{0.8}$

$1.6 \div 2$
16 tenths $\div 2$
8 tenths
0.8



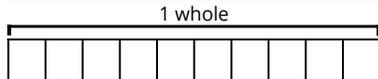
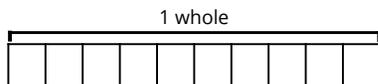
3. Then complete this one.

Solve.

$2.7 \div 3 = \underline{\hspace{2cm}}$

2. Answer these questions.

1. Shade the model below to represent 1.6.



2. Using the model above, explain how you know that 16 tenths is the same as 1.6.

3. Why was it helpful for Xavier to make 1.6 into 16 tenths in the step marked with an arrow?

Decimals

Dividing Decimals Using Place Value

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to use place value to divide decimal numbers.

- 1. Students study Xavier's work. Xavier has completed his work correctly.**
- 2. Students respond to three questions related to Xavier's work:**

1. Shade the model below to represent 1.6.



2. Using the model above, explain how you know that 16 tenths is the same as 1.6.

Sample response:

I know they are the same because one whole and 6 of the 10 tenths are shaded. 16 tenths total are shaded.

3. Why was it helpful for Xavier to make 1.6 into 16 tenths in the step marked with an arrow?

Sample responses:

16 is easily divisible by 2.

He knows that 16 divided by 2 equals 8.

- 3. Students do a similar problem on their own.**

Solution:

0.9

Teacher Notes

SET 2 OF 2

TARGET:

*Understanding how to use place value to divide decimal numbers.
(Targeted Error: Misunderstanding scale when dividing by a larger number.)*

1. Students study Abigail's work. Abigail has completed her work incorrectly.

2. Students respond to two questions related to Abigail's work:

- Abigail was right that 40 is 10 times more than 4, but this does not mean that the answer is 10 times more. Why not?

Sample responses:

We are dividing 60.12 by a number that is 10 times more so the answer will be a smaller number.

The answer is 10 times less because he is dividing the same amount by a larger number, so the answer will be smaller.

- What should the answer be?

1.503

3. Students do a similar problem on their own.

Solution:

2.305

Sample explanation:

Instead of 3 groups there are 30 groups. That's 10 times more groups, so there must be 10 times less in each group.

SET TWO

1. Study Abigail's incorrect work.

X

Name: Abigail

Use place value reasoning and the first quotient to figure out the second quotient. Explain your thinking.

$60.12 \div 4 = 15.03$

$60.12 \div 40 = \underline{150.3}$

Explanation:
 Instead of 4 it's 40.
 That's 10 times more,
 so the answer must
 be 10 times more.



2. Answer these questions.

- Abigail was right that 40 is 10 times more than 4, but this does not mean that the answer is 10 times more. Why not?

- What should the answer be?

3. Then complete this one.

Use place value reasoning and the first quotient to figure out the second quotient. Explain your thinking.

$$69.15 \div 3 = 23.05$$

Explanation:

$$69.15 \div 30 = \underline{\hspace{2cm}}$$

Name: _____ Date: _____

SET ONE

1. Study Daniel's incorrect work.

X Name: Daniel

Solve.

$$138 \div 5 = 27.3$$
$$\begin{array}{r} 27.3 \\ 5 \overline{)138} \\ \underline{10} \\ 38 \\ \underline{35} \\ 3 \end{array}$$

3. Then complete this one.

Solve.

$$113 \div 2 = \underline{\hspace{2cm}}$$

2. Answer these questions.

- 1. Daniel was not correct in writing the remainder of 3 as 3 tenths in the quotient. Why is the remainder of 3 not equivalent to 3 tenths?

- 2. How could Daniel have decomposed his remainder of 3 to continue dividing?

Decimals

Dividing Decimals

Teacher Notes

SET 1 OF 2

TARGET:

Misbelieving that a remainder and a decimal quotient are the same. (Targeted Error: Carrying over the remaining value without considering the divisor.)

1. Students study Daniel's work. Daniel has completed his work incorrectly.

2. Students respond to two questions related to Daniel's work:

- 1. Daniel was not correct in writing the remainder of 3 as 3 tenths in the quotient. Why is the remainder of 3 not equivalent to 3 tenths?

Sample response:

A remainder of 3 is 3 ones, not 3 tenths.

- 2. How could Daniel have decomposed his remainder of 3 to continue dividing?

Sample response:

Daniel could have written 3 ones as 30 tenths to continue dividing by 5.

3. Students do a similar problem on their own.

Solution:

$$56.5$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to represent a division word problem using a tape diagram.

1. Students study Madison's work. Madison has completed her work correctly.

2. Students respond to two questions related to Madison's work:

- Why did Madison split her diagram into 6 sections?

Sample responses:

She was dividing by 6.

The woman was buying 6 calculators.

She had to divide 50.16 into 6 equal parts.

- What do each of the 6 sections represent?

Sample response:

the cost of each calculator

3. Students do a similar problem on their own.

Solution:

Each comic book was \$4.31.

SET TWO

1. Study Madison's correct work.

Name: Madison

Solve. Write your answer in sentence form.

A woman spent \$50.16 buying 6 of the same calculator. How much did each calculator cost?

$$\begin{array}{r} 8.36 \\ 6 \overline{) 50.16} \\ \underline{-48} \\ 21 \\ \underline{-18} \\ 30 \\ \underline{-36} \\ 0 \end{array}$$

Each calculator cost \$8.36

2. Answer these questions.

- Why did Madison split her diagram into 6 sections?
- What do each of the 6 sections represent?

3. Then complete this one.

Solve. Write your answer in sentence form.

A girl spent \$34.48 buying 8 new comic books. If each cost the same amount, how much was each comic book?

SET ONE

1. Study Charlotte's incorrect work.

X

Name: Charlotte

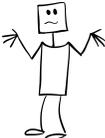
Convert grams to kilograms. Remember!
1 kg = 1,000 g

60.1 g = 60,100 kg

= $60.1 \times (1g)$

= $60.1 \times (1000kg)$ ←

= 60,100 kg



3. Then complete this one.

Convert grams to kilograms.

Remember!
1 kg = 1,000 g

41.8 g = _____ kg

2. Answer these questions.

- 1. Are kilograms larger or smaller than grams? Explain how you know.

- 2. Based on your answer above, should Charlotte's number of kilograms be more or less than 60.1? Explain how you know.

- 3. What should Charlotte change in the step marked with an arrow in order to solve the problem correctly?

Teacher Notes

SET 1 OF 2

TARGET:

Understanding scale when converting from one unit to another. (Targeted Error: Using the wrong scale factor.)

1. Students study Charlotte's work. Charlotte has completed her work incorrectly.

2. Students respond to three questions related to Charlotte's work:

- 1. Are kilograms larger or smaller than grams? Explain how you know.

Sample response:

Larger, 1 kilogram is equal to 1,000 grams.

- 2. Based on your answer above, should Charlotte's number of kilograms be more or less than 60.1? Explain how you know.

Sample response:

Less than 60.1, kilograms are larger than grams.

- 3. What should Charlotte change in the step marked with an arrow in order to solve the problem correctly?

Sample responses:

She should multiply by 0.001 kg.

She should divide by 1,000 kg.

3. Students do a similar problem on their own.

Solution:

0.0418 kg

Teacher Notes

SET 2 OF 2

TARGET:

Understanding when to multiply or divide using the conversion factor.

1. Students study Benjamin's work. Benjamin has completed his work correctly.

2. Students respond to a question related to Benjamin's work:

- 🗣️ In the step marked with an arrow, how did Benjamin know to multiply by 100 instead of divide by 100?

Sample response:

Meters are larger than centimeters, so you need more centimeters to make up a meter.

3. Students do a similar problem on their own.

Solution:

625 cm

SET TWO

1. Study Benjamin's correct work.

✓ Name: Benjamin

Convert meters to centimeters. Remember!
1 m = 100 cm

2.25 m = 225 cm

= 2.25 × (1m)

= 2.25 × (100 cm) ←

= 225 cm



2. Answer the question.

- 🗣️ In the step marked with an arrow, how did Benjamin know to multiply by 100 instead of divide by 100?

3. Then complete this one.

Convert meters to centimeters.

Remember!
1 m = 100 cm

6.25 m = _____ cm

Name: _____ Date: _____

SET ONE

1. Study Christian's incorrect work.

X Name: Christian

Solve. Write your answer in sentence form.

Remember!
1 kg = 1,000 g

When the children weighed all five of their puppies together, they weighed 28.35 kg. Since all of the puppies are about the same size, about how many grams does each puppy weigh?

$$\begin{array}{r} 5.67 \\ 5 \overline{)28.35} \\ \underline{-25} \\ 33 \\ \underline{-30} \\ 35 \\ \underline{-35} \\ 0 \end{array}$$

Each puppy weighs about 5.67 kg.



2. Answer these questions.

- 1. Christian is correct that each puppy weighs about 5.67 kg. However, what unit was Christian supposed to use in his final answer?
- 2. How should Christian convert from kilograms to grams?

3. Then complete this one.

Solve. Write your answer in sentence form.

Remember!
1 km = 1,000 m

A man swam 3.88 kilometers over the course of 4 weeks. He swam the same distance each week. How many meters did he swim each week?

Decimals

Converting Units Word Problems

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to interpret a word problem involving unit conversion. (Targeted Error: Forgetting to convert the units when a word problem asks for a different unit than what is provided.)

1. **Students study Christian's work. Christian has completed his work incorrectly.**
2. **Students respond to two questions related to Christian's work:**

- 1. Christian is correct that each puppy weighs about 5.67 kg. However, what unit was Christian supposed to use in his final answer?
grams

- 2. How should Christian convert from kilograms to grams?

Sample response:

He should multiply 5.67 by 1000.

3. **Students do a similar problem on their own.**

Solution:

He swam 970 meters each week.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the order of operations in the context of solving word problems.

1. Students study Harper's work. Harper has completed her work correctly.

2. Students respond to a question related to Harper's work:

- First, Harper found out how many total milliliters of juice were needed. Then, she converted to liters. Could she have converted first? Explain why or why not.

Sample response:

Yes, because with the order of operations you can multiply and divide in any order.

3. Students do a similar problem on their own.

Solution:

She needs 3.6 liters of vegetable broth for 4 recipes of soup.

SET TWO

1. Study Harper's correct work.



Solve. Write your answer in sentence form.

Remember!
1 L = 1,000 mL

 Name: Harper

A boy is making punch. One recipe uses 700 milliliters of juice. How many liters of juice does he need for 6 recipes of punch?

1 recipe = 700 mL 4200 mL = _____ Liters
6 recipes = 6 × 700 mL = 4200 ÷ 1000 Liters
 = 4200 mL = 4.2 Liters

He needs 4.2 liters of juice.



2. Answer the question.

- First, Harper found out how many total milliliters of juice were needed. Then, she converted to liters. Could she have converted first? Explain why or why not.

3. Then complete this one.

Solve. Write your answer in sentence form.

Remember!
1 L = 1,000 mL

A girl is making soup. For each recipe of soup she needs 900 milliliters of vegetable broth. How many liters of vegetable broth does she need for 4 recipes of soup?

Name: _____ Date: _____

SET ONE

1. Study Gavin's incorrect work.

X

Name: Gavin

Fill in the boxes to make the number sentences true.

$$5 \div 10 = \frac{\boxed{10}}{\boxed{5}}$$
$$\frac{5}{10} = \boxed{10} \div \boxed{5}$$


2. Answer these questions.

- 1. Gavin decided to switch the 5 and 10 because he thought you are always supposed to divide the larger number by the smaller number. Is this true?

- 2. Draw a diagram to represent $5 \div 10$.

- 3. When you divide a smaller whole number by a larger whole number, will the answer be greater than 1 or less than 1?

Basic Operations with Fractions
Fractions as Division

Teacher Notes

SET 1 OF 2

TARGET:

*Understanding how division number sentences relate to fractions.
(Targeted Error: Dividing the larger number by the smaller number, regardless of order.)*

1. Students study Gavin's work. Gavin has completed his work incorrectly.

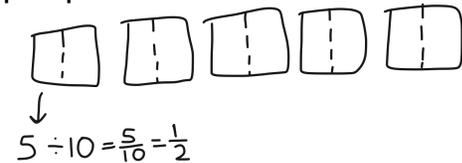
2. Students respond to three questions related to Gavin's work:

- 1. Gavin decided to switch the 5 and 10 because he thought you are always supposed to divide the larger number by the smaller number. Is this true?

No, it is not true.

- 2. Draw a diagram to represent $5 \div 10$.

Sample response:



- 3. When you divide a smaller whole number by a larger whole number, will the answer be greater than 1 or less than 1?

Sample response:

less than one.

3. Students do a similar problem on their own.

Solution:

$$\frac{6}{18} \text{ or } \frac{1}{3}$$

$$6 \div 18$$

3. Then complete this one.

Fill in the boxes to make the number sentences true.

$$6 \div 18 = \frac{\boxed{}}{\boxed{}}$$

$$\frac{6}{18} = \boxed{} \div \boxed{}$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding that multiple diagrams can be used to model a division problem.

1. Students study Samira's work. Samira has completed her work correctly.

2. Students respond to a question related to Samira's work:

- Samira drew two different pictures but got the same answer. How do both pictures show $3 \div 2$?

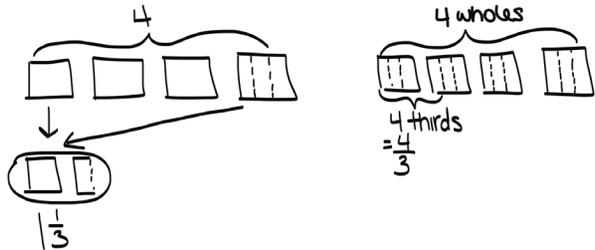
Sample response:

The first one shows 3 wholes together divided by 2, which becomes $\frac{3}{2} = 1\frac{1}{2}$ and the second shows each of the 3 wholes divided by 2, which equals 3 halves.

3. Students do a similar problem on their own.

Sample solutions:

$$1\frac{1}{3} \text{ or } \frac{4}{3}$$



SET TWO

1. Study Samira's correct work.

✓

Name: Samira

Draw **two** different pictures to represent the expression. Express your answer as a fraction.

$3 \div 2$

3

$1\frac{1}{2}$

3 wholes

3 halves $\frac{3}{2} = 1\frac{1}{2}$

2. Answer the question.

- Samira drew two different pictures but got the same answer. How do both pictures show $3 \div 2$?

3. Then complete this one.

Draw **two** different pictures to represent the expression. Express your answer as a fraction.

$$4 \div 3$$

Name: _____ Date: _____

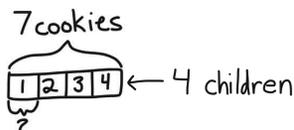
SET ONE

1. Study Rob's correct work.

 Name: Rob

Solve. Express your answer as a fraction. Write your answer in sentence form.

The teacher would like to share 7 cookies equally among 4 children. How many cookies will each child receive?



$7 \div 4 = \frac{7}{4} = 1\frac{3}{4}$

Each child will receive $1\frac{3}{4}$ cookies.



2. Answer the question.

- Based on the word problem, why did Rob divide by 4 instead of multiply by 4?

3. Then complete this one.

Solve. Express your answer as a fraction. Write your answer in sentence form.

The cook used 8 pounds of potatoes in 12 pots of soup. If each pot had the same amount of potatoes, how many pounds of potatoes did he use in each pot?

Basic Operations with Fractions
Fraction as Division Word Problems

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to interpret a word problem for division. (Targeted Error: Using the wrong operation.)

1. Students study Rob's work. Rob has completed his work correctly.

2. Students respond to a question related to Rob's work:

- Based on the word problem, why did Rob divide by 4 instead of multiply by 4?

Sample response:

When you are sharing something equally, it means taking what you have and dividing it into equal parts.

3. Students do a similar problem on their own.

Solution:

The cook used $\frac{2}{3}$ of a pound of potatoes in each pot.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to interpret a word problem for division. (Targeted Error: Misinterpreting a division word problem and switching the dividend and divisor.)

1. Students study Serenity's work. Serenity has completed her work incorrectly.

2. Students respond to two questions related to Serenity's work:

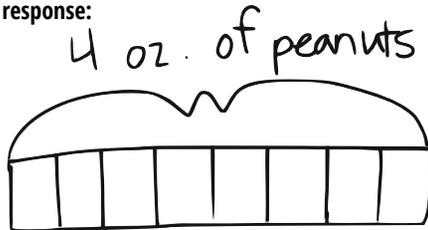
- Based on the word problem, how can you tell that Serenity should have solved $4 \div 8$, rather than $8 \div 4$?

Sample response:

The waiter has a total of 4 oz, and he needs to divide it among 8 sundaes.

- Draw a diagram to represent $4 \div 8$.

Sample response:



3. Students do a similar problem on their own.

Solution:

She has to run $1\frac{3}{7}$ miles each day.

SET TWO

1. Study Serenity's incorrect work.

X Name: Serenity

Solve. Express your answer as a fraction. Write your answer in sentence form.

A waiter uses 4 ounces of peanuts to make 8 ice cream sundaes. If each sundae has the same amount of peanuts, how many ounces of peanuts does he use on each sundae?

8 sundaes

$8 \div 4 = \frac{8}{4} = 2$

The waiter uses 2 ounces on each sundae.

2. Answer these questions.

- Based on the word problem, how can you tell that Serenity should have solved $4 \div 8$, rather than $8 \div 4$?
- Draw a diagram to represent $4 \div 8$.

3. Then complete this one.

Solve. Express your answer as a fraction. Write your answer in sentence form.

The soccer player wants to run a total of 10 miles in 7 days. If she runs the same amount each day, how many miles does she run each day?

Name: _____ Date: _____

SET ONE

1. Study Mateo's correct work.

Name: MATEO

Solve.

$$\frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$

$\frac{2}{3} = \frac{4}{6}$

$\frac{1}{2} = \frac{3}{6}$

$$\frac{4}{6} + \frac{3}{6} = \frac{7}{6}$$

2. Answer the question.

- Why did Mateo find equivalent fractions for $\frac{2}{3}$ and $\frac{1}{2}$?

3. Then complete this one.

Solve.

$$\frac{2}{5} + \frac{1}{2} =$$

Basic Operations with Fractions
Adding Fractions with Unlike Denominators

Teacher Notes

SET 1 OF 2

TARGET:

Understanding why equivalent fractions with a common denominator must be found before adding fractions.

1. Students study Mateo's work. Mateo has completed his work correctly.

2. Students respond to a question related to Mateo's work:

- Why did Mateo find equivalent fractions for $\frac{2}{3}$ and $\frac{1}{2}$?

Sample response:

You cannot add fractions of unequal parts. They must have the same denominator in order to add them together.

3. Students do a similar problem on their own.

Solution:

$$\frac{9}{10}$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding that in order to add two fractions they must have a common denominator. (Targeted Error: Adding the denominators together.)

1. Students study Alexa's work. Alexa has completed her work incorrectly.

2. Students respond to three questions related to Alexa's work:

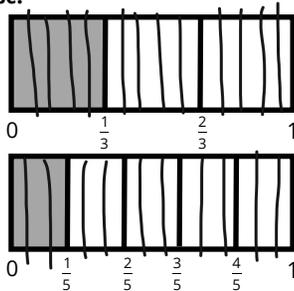
- What mistake did Alexa make when adding the fractions?

Sample response:

She added denominators together without finding a common denominator.

- Help Alexa by partitioning the diagrams below into an equal number of parts in order to find a common denominator.

Sample response:



- Based on the diagrams above, what fraction is $\frac{1}{3}$ equivalent to?

$\frac{5}{15}$

3. Students do a similar problem on their own.

Solution:

$\frac{7}{12}$

SET TWO

1. Study Alexa's incorrect work.

X

Name: Alexa

Solve.

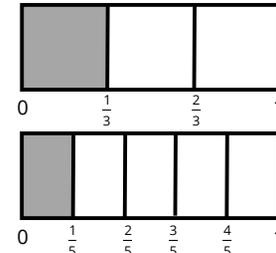
$$\frac{1}{3} + \frac{1}{5} = \frac{2}{8}$$

$$\frac{1+1}{3+5} = \frac{2}{8}$$

2. Answer these questions.

- What mistake did Alexa make when adding the fractions?

- Help Alexa by partitioning the diagrams below into an equal number of parts in order to find a common denominator.



- Based on the diagrams above, what fraction is $\frac{1}{3}$ equivalent to?

3. Then complete this one.

Solve.

$$\frac{1}{3} + \frac{1}{4} =$$

Name: _____ Date: _____

SET ONE

1. Study Laura's correct work.

 Solve. Write your answer in sentence form.

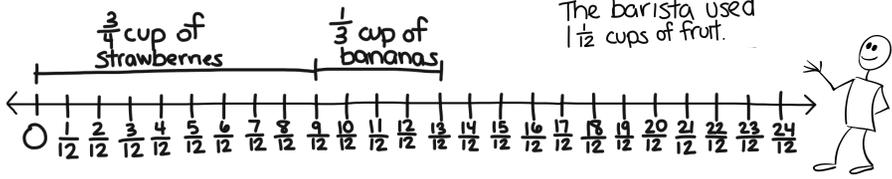
Name: Laura

A barista made a fruit smoothie with $\frac{3}{4}$ of a cup of strawberries and $\frac{1}{3}$ of a cup of bananas. How much fruit did the barista use in all?

$\frac{3}{4} + \frac{1}{3}$
 $\frac{9}{12} + \frac{4}{12}$
 $\frac{13}{12}$
 $1\frac{1}{12}$

$\frac{3}{4}$ cup of strawberries $\frac{1}{3}$ cup of bananas

The barista used $1\frac{1}{12}$ cups of fruit.



2. Answer the question.

- How does Laura know that $\frac{3}{4}$ is equivalent to $\frac{9}{12}$?

3. Then complete this one.

Solve. Write your answer in sentence form.

A cook made a stew with $\frac{5}{6}$ of a cup of broccoli and $\frac{1}{2}$ of a cup of carrots. How many cups of vegetables did the cook use in all?

Teacher Notes

SET 1 OF 2

TARGET:

Understanding why two different fractions can have the same value.

1. Students study Laura's work. Laura has completed her work correctly.

2. Students respond to a question related to Laura's work:

- How does Laura know that $\frac{3}{4}$ is equivalent to $\frac{9}{12}$?

Sample response:

If you multiply $\frac{3}{4}$ by $\frac{3}{3}$, you get $\frac{9}{12}$.

3. Students do a similar problem on their own.

Solution:

The cook used $\frac{8}{6}$ (or $1\frac{1}{3}$) cups of vegetables.

Teacher Notes

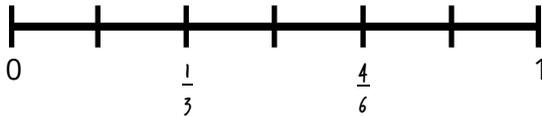
SET 2 OF 2

TARGET:

Understanding how to find common denominators when adding fractions. (Targeted Error: Adding, instead of multiplying, the same number to the numerator and denominator in order to find equivalent fractions.)

- Students study Brayden's work. Brayden has completed his work incorrectly.
- Students respond to two questions related to Brayden's work:

- Look at the step marked with an arrow. Show Brayden that $\frac{1}{3}$ and $\frac{4}{6}$ do not have the same value by marking them on the number line.



- How should Brayden have found a fraction equivalent to $\frac{1}{3}$ with a denominator of 6?

Sample responses:

He should have multiplied both the numerator and denominator by 2.

He should have multiplied $\frac{1}{3}$ by $\frac{2}{2}$.

- Students do a similar problem on their own.

Solution:

The teenager spent $\frac{11}{12}$ of her savings.

SET TWO

- Study Brayden's incorrect work.

X

Name: Brayden

Solve. Write your answer in sentence form.

A 5th grader spent $\frac{1}{3}$ of his allowance on movie tickets and $\frac{1}{6}$ on gum. What fraction of his allowance did he spend?

allowance

$\frac{1}{3}$ movie ticket	$\frac{1}{6}$ gum	
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?

$$\frac{1}{3} + \frac{1}{6}$$

$$\frac{1+3}{3+3} + \frac{1}{6}$$

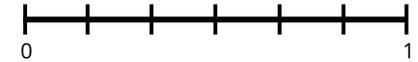
$$\frac{4}{6} + \frac{1}{6}$$

$$\frac{5}{6}$$

He spent $\frac{5}{6}$ of his allowance.

- Answer these questions.

- Look at the step marked with an arrow. Show Brayden that $\frac{1}{3}$ and $\frac{4}{6}$ do not have the same value by marking them on the number line.



- How should Brayden have found a fraction equivalent to $\frac{1}{3}$ with a denominator of 6?

- Then complete this one.

Solve. Write your answer in sentence form.

A teenager spent $\frac{2}{3}$ of her savings on a new outfit and $\frac{1}{4}$ on new shoes. What fraction of her savings did she spend?

Name: _____ Date: _____

SET ONE

1. Study Nevaeh's correct work.

✓ Name: Nevaeh

Solve.

$$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

The first tape diagram shows a number line from 0 to 1 with 12 equal parts. The first 4 parts are shaded to represent $\frac{1}{3}$. The next 3 parts are marked with 'x' and circled, representing the $\frac{1}{4}$ being subtracted. The remaining 1 part is shaded, representing the result $\frac{1}{12}$. The second tape diagram shows a number line from 0 to 1 with 4 equal parts, with the first part shaded to represent $\frac{1}{4}$. A small cartoon character is drawn to the right of the diagrams.

2. Answer these questions.

- 1. Why did Nevaeh partition both diagrams into 12 equal parts?
- 2. Why did Nevaeh cross off 3 parts in the first diagram?

3. Then complete this one.

Solve.

$$\frac{2}{3} - \frac{1}{2} =$$

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to use tape diagrams to find equivalent fractions and solve a subtraction fraction problem.

1. Students study Nevaeh's work. Nevaeh has completed her work correctly.

2. Students respond to two questions related to Nevaeh's work:

- 1. Why did Nevaeh partition both diagrams into 12 equal parts?

Sample response:

She needed to divide the diagrams into the same number of parts.

- 2. Why did Nevaeh cross off 3 parts in the first diagram?

Sample response:

She needs to subtract $\frac{3}{12}$, which is the same as $\frac{1}{4}$.

3. Students do a similar problem on their own.

Solution:

$$\frac{1}{6}$$

Basic Operations with Fractions
Subtracting Fractions with Unlike Denominators, continued

Teacher Notes

SET 2 OF 2

TARGET:

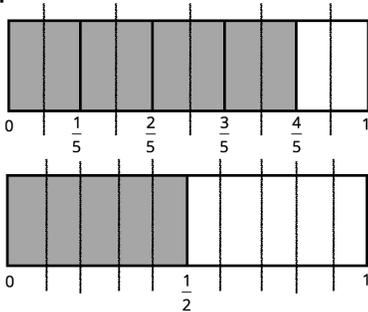
Understanding how to use tape diagrams to find equivalent fractions and solve a subtraction fraction problem. (Targeted Error: Subtracting the numerators and using the denominator in the first fraction in the answer when subtracting fractions.)

1. Students study Colton's work. Colton has completed his work incorrectly.

2. Students respond to two questions related to Colton's work:

- Colton forgot to find common denominators before subtracting. Help Colton partition the diagrams below into the correct number of parts.

Sample response:



- Based on your partitioning of the diagram above, what denominator should Colton have used? Explain how you know.

Sample response:

Colton should have used tenths because both fifths and halves can be divided equally into tenths.

3. Students do a similar problem on their own.

Solution:

$$\frac{2}{15}$$

Basic Operations with Fractions
Subtracting Fractions with Unlike Denominators, continued

SET TWO

1. Study Colton's incorrect work.

X

Name: Colton

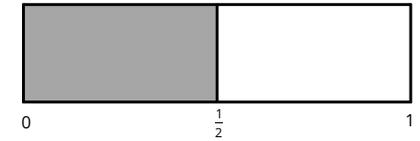
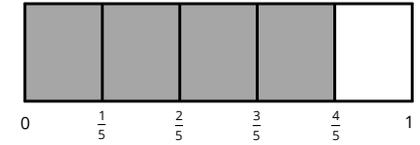
Solve.

$$\frac{4}{5} - \frac{1}{2} = \frac{3}{5}$$

$$4 - 1 = 3$$


2. Answer these questions.

- Colton forgot to find common denominators before subtracting. Help Colton partition the diagrams below into the correct number of parts.



- Based on your partitioning of the diagram above, what denominator should Colton have used? Explain how you know.

3. Then complete this one.

Solve.

$$\frac{1}{3} - \frac{1}{5} =$$

SET ONE

1. Study Stella's incorrect work.

X

Name: Stella

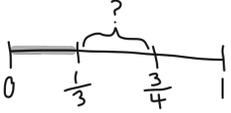
Solve. Write your answer in sentence form.

The student's run to school is $\frac{3}{4}$ of a mile. She has run $\frac{1}{3}$ of a mile so far. How far does she still have to run before getting to school?

$$\frac{3}{4} \times 3 - \frac{1}{3} \times 4$$

$$\frac{3}{12} - \frac{1}{12}$$

$$\frac{2}{12}$$



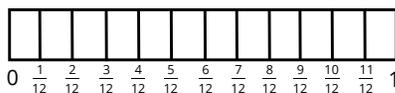
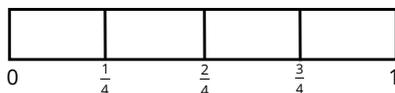
She still has to run $\frac{2}{12}$ of a mile.



2. Answer these questions.

- ☉ Stella correctly found common denominators. What did she forget to do next to create equivalent fractions?

- ☉ Shade the tape diagrams below to show that $\frac{3}{4}$ and $\frac{3}{12}$ are not equivalent.



- ☉ Which fraction with a denominator 12 is equivalent to $\frac{3}{4}$? Explain how you know.

3. Then complete this one.

Solve. Write your answer in sentence form.

A 5th grader's walk home is $\frac{1}{2}$ of a mile. He has walked $\frac{1}{5}$ of a mile so far. How far does he still have to walk before getting home?

Basic Operations with Fractions

Subtraction Word Problems

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to find common denominators in order to subtract fractions. (Targeted Error: Multiplying the denominators to find a common denominator without multiplying the numerators to find equivalent fractions.)

1. Students study Stella's work. Stella has completed her work incorrectly.

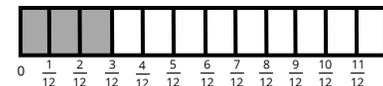
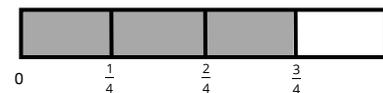
2. Students respond to three questions related to Stella's work:

- ☉ Stella correctly found common denominators. What did she forget to do next to create equivalent fractions?

Sample response:

She forgot to change the numerators.

- ☉ Shade the tape diagrams below to show that $\frac{3}{4}$ and $\frac{3}{12}$ are not equivalent.



- ☉ Which fraction with a denominator 12 is equivalent to $\frac{3}{4}$? Explain how you know.

Sample response:

$\frac{9}{12}$, because they both take up the same part of the whole.

3. Students do a similar problem on their own.

Solution:

He still has to walk $\frac{3}{10}$ of a mile before getting home.

Teacher Notes

SET 2 OF 2

TARGET:

Interpreting a word problem that includes more than two numbers.

1. Students study Josiah's work. Josiah has completed his work correctly.

2. Students respond to a question related to Josiah's work:

- In Josiah's subtraction problem, why did he use three numbers, rather than just two?

Sample response:

He started with 1 pound of flour and used it twice, not just once.

3. Students do a similar problem on their own.

Solution:

The cook has $\frac{2}{6}$ of a carton left.

SET TWO

1. Study Josiah's correct work.

✓ Name: Josiah

Solve. Write your answer in sentence form.

The baker had 1 pound of flour. He used $\frac{1}{6}$ of a pound to bake a cake and $\frac{1}{3}$ of a pound to make a batch of cupcakes. What fraction of the flour does he have left?

$$\begin{array}{r} 1 - \frac{1}{6} - \frac{1}{3} \\ \frac{6}{6} - \frac{1}{6} - \frac{2}{6} \\ \hline \frac{5}{6} - \frac{2}{6} \\ \hline \frac{3}{6} \end{array}$$

He has $\frac{3}{6}$ of a pound left.

2. Answer the question.

- In Josiah's subtraction problem, why did he use three numbers, rather than just two?

3. Then complete this one.

Solve. Write your answer in sentence form.

The cook had 1 carton of eggs. She used $\frac{1}{2}$ of the carton to make scrambled eggs and $\frac{1}{6}$ of the carton to make pancakes. What fraction of the carton does she have left?

Name: _____ Date: _____

SET ONE

1. Study Brandon's correct work.

Name: Brandon

Using mental math, determine whether the expression is greater than 1 or less than 1. Explain your reasoning.

$$\frac{3}{4} + \frac{1}{7}$$

Greater than 1
Less than 1

Explanation:
I know that the second fraction needs to be greater than $\frac{1}{4}$ in order for the expression to be greater than 1 because $\frac{3}{4} + \frac{1}{4} = 1$ or 1. $\frac{1}{7}$ is less than $\frac{1}{4}$.

2. Answer the question.

- Using Brandon's reasoning, would the expression $\frac{3}{4} + \frac{1}{3}$ be greater than 1 or less than 1? Explain your reasoning.

3. Then complete this one.

Using mental math, determine whether the expression is greater than 1 or less than 1. Explain your reasoning.

$$\frac{3}{5} + \frac{1}{2}$$

Explanation:

Greater than 1

Less than 1

Basic Operations with Fractions
Using Benchmark Fractions to Estimate Answers (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to use 1 as a benchmark to estimate the answer.

1. Students study Brandon's work. Brandon has completed his work correctly.

2. Students respond to a question related to Brandon's work:

- Using Brandon's reasoning, would the expression $\frac{3}{4} + \frac{1}{3}$ be greater than 1 or less than 1? Explain your reasoning.

Sample response:

Greater than 1, $\frac{1}{3}$ is larger than $\frac{1}{4}$, and $\frac{1}{4} + \frac{3}{4} = 1$.

3. Students do a similar problem on their own.

Solution:

greater than 1

Sample explanation:

I know that $\frac{1}{2}$ needs to be greater than $\frac{2}{5}$ because $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ and $\frac{3}{5} = 1$. $\frac{1}{2}$ is greater than $\frac{2}{5}$.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to correctly use $\frac{1}{2}$ as a benchmark to estimate the answer.

1. Students study Mia's work. Mia has completed her work incorrectly.

2. Students respond to three questions related to Mia's work:

- Mia is correct that $\frac{5}{8}$ is greater than $\frac{1}{2}$. How far is $\frac{5}{8}$ from $\frac{1}{2}$? Explain.

Sample response:

$$\frac{1}{8}. \text{ Since } \frac{4}{8} = \frac{1}{2}, \frac{5}{8} - \frac{4}{8} = \frac{1}{8}.$$

- Is $\frac{3}{16}$ greater or less than your answer above? Explain.

Sample response:

$\frac{3}{16}$ is greater than $\frac{1}{8}$ because $\frac{1}{8}$ is equal to $\frac{2}{16}$, and $\frac{3}{16}$ is greater than $\frac{2}{16}$.

- Based on your answers above, how do you know that $\frac{5}{8} - \frac{3}{16}$ is actually **less** than $\frac{1}{2}$?

Sample response:

I know the second fraction needs to be greater than $\frac{1}{8}$ for the expression to be less than $\frac{1}{2}$. $\frac{3}{16}$ is greater than $\frac{1}{8}$.

3. Students do a similar problem on their own.

Solution:

less than $\frac{1}{2}$

Sample explanation:

$\frac{5}{6}$ is $\frac{2}{6}$ away from $\frac{1}{2}$, since $\frac{3}{6}$ equals $\frac{1}{2}$. $\frac{5}{6} - \frac{3}{6} = \frac{2}{6}$. Therefore $\frac{2}{3}$ must be greater than $\frac{2}{6}$ in order for the expression to be less than $\frac{1}{2}$. $\frac{2}{6}$ is the same as $\frac{1}{3}$. $\frac{2}{3}$ is greater than $\frac{1}{3}$.

SET TWO

1. Study Mia's incorrect work.

X

Name: Mia

Using mental math, determine whether the expression is greater than $\frac{1}{2}$ or less than $\frac{1}{2}$. Explain your reasoning.

$$\frac{5}{8} - \frac{3}{16}$$

Greater than $\frac{1}{2}$
Less than $\frac{1}{2}$

Explanation:

I know that the expression is greater than $\frac{1}{2}$ because $\frac{5}{8}$ is already greater than $\frac{1}{2}$ and you are only subtracting a very little bit from it.



2. Answer these questions.

- Mia is correct that $\frac{5}{8}$ is greater than $\frac{1}{2}$. How far is $\frac{5}{8}$ from $\frac{1}{2}$? Explain.

- Is $\frac{3}{16}$ greater or less than your answer above? Explain.

- Based on your answers above, how do you know that $\frac{5}{8} - \frac{3}{16}$ is actually **less** than $\frac{1}{2}$?

3. Then complete this one.

Using mental math, determine whether the expression is greater than $\frac{1}{2}$ or less than $\frac{1}{2}$. Explain your reasoning.

$$\frac{5}{6} - \frac{2}{3}$$

Explanation:

Greater than $\frac{1}{2}$

Less than $\frac{1}{2}$

Name: _____ Date: _____

SET ONE

1. Study Autumn's correct work.



Name: Autumn

Using mental math, make the statement true by using the less than or greater than symbol. Explain your reasoning.

$$5\frac{2}{3} + 2\frac{1}{8} \boxed{<} 8$$

Explanation:
The expression on the left is less than 8 because $5+2=7$, therefore $\frac{2}{3} + \frac{1}{8}$ needs to be greater than 1 for the left side to be greater than 8. However, it is not since $\frac{1}{8}$ is less than $\frac{1}{3}$.



2. Answer the question.

- Read Autumn's explanation. Describe another way you could use mental math to explain why $5\frac{2}{3} + 2\frac{1}{8}$ is less than 8.

3. Then complete this one.

Using mental math, make the statement true by using the less than or greater than symbol. Explain your reasoning.

$$3\frac{1}{7} + 3\frac{11}{12} \boxed{\phantom{<}} 7$$

Explanation:

Basic Operations with Fractions
Using Benchmark Fractions to Estimate Answers (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that there are multiple ways to use mental math to compare fractions in different forms.

1. Students study Autumn's work. Autumn has completed her work correctly.

2. Students respond to a question related to Autumn's work:

- Read Autumn's explanation. Describe another way you could use mental math to explain why $5\frac{2}{3} + 2\frac{1}{8}$ is less than 8.

Sample response:

The expression on the left is less than 8 because $\frac{2}{3} + \frac{1}{8}$ needs to be greater than 1 for the left side to be greater than 8. However, it is not since $\frac{2}{3}$ is less than $\frac{7}{8}$. $\left(\frac{1}{8} + \frac{7}{8} = 1\right)$.

3. Students do a similar problem on their own.

Solution:



Sample explanation:

The expression on the left is greater because $3 + 3 = 6$, therefore $\frac{1}{7} + \frac{11}{12}$ must be greater than 1 for that expression to be greater than 7. $\frac{11}{12} + \frac{1}{12} = 1$. So $\frac{1}{7}$ must be larger than $\frac{1}{12}$, which it is.

Teacher Notes

SET 2 OF 2

TARGET:

Using mental math to compare fractions in different forms. (Targeted Error: Believing that all fractions are smaller than mixed numbers, regardless of value.)

1. Students study Tyler's work. Tyler has completed his work incorrectly.

2. Students respond to four questions related to Tyler's work:

1. Explain why fractions are **not always** less than 1.

Sample response:

You can have more parts than it takes to fill a whole.

2. How can you tell that $\frac{22}{2}$ is **greater** than 1?

Sample response:

The value of the numerator is more than the value of the denominator.

3. What whole number has the same value as $\frac{22}{2}$?

11

4. Based on your answer above, how do you know that $8\frac{1}{2} - 1\frac{3}{4}$ is **less** than $\frac{22}{2}$?

Sample response:

$8 - 1 = 7$, and 7 is less than 11.

3. Students do a similar problem on their own.

Solution:

<

Sample explanation:

$\frac{20}{4}$ is equal to 5. And $7 - 4$ is 3, so the expression is less than $\frac{20}{4}$.

SET TWO

1. Study Tyler's incorrect work.

X Name: Tyler

Using mental math, make the statement true by using the less than or greater than symbol. Explain your reasoning.

$$8\frac{1}{2} - 1\frac{3}{4} \boxed{>} \frac{22}{2}$$

Explanation:

$8\frac{1}{2} - 1\frac{3}{4}$ is greater than $\frac{22}{2}$ because mixed numbers are always greater than 1 and fractions are always less than 1.



2. Answer these questions.

1. Explain why fractions are **not always** less than 1.
2. How can you tell that $\frac{22}{2}$ is **greater** than 1?
3. What whole number has the same value as $\frac{22}{2}$?
4. Based on your answer above, how do you know that $8\frac{1}{2} - 1\frac{3}{4}$ is **less** than $\frac{22}{2}$?

3. Then complete this one.

Using mental math, make the statement true by using the less than or greater than symbol. Explain your reasoning.

$$7\frac{1}{8} - 4\frac{1}{3} \boxed{} \frac{20}{4}$$

Explanation:

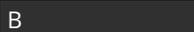
Name: _____ Date: _____

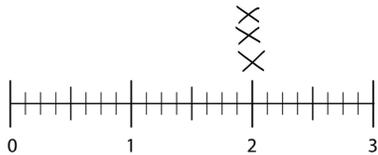
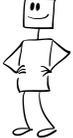
SET ONE

1. Study Allie's correct work.

 Name: Allie

Using the ruler, measure each rod to the **nearest inch**. Complete the line plot using these measurements.

A 
B 
C 

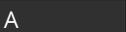
2. Answer these questions.

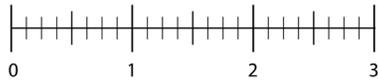
Looking at the ruler on the left, which rods (A, B, C) do not fall directly on the 2 inch marker?

Measuring to the nearest inch is not the most precise way to measure these three rods. If you wanted a more precise measurement, what could you do?

3. Then complete this one.

Using the ruler, measure each rod to the **nearest inch**. Complete the line plot using these measurements.

A 
B 
C 

Basic Operations with Fractions
Fraction Line Plots (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to measure more or less precisely.

1. Students study Allie's work. Allie has completed her work correctly.

2. Students respond to two questions related to Allie's work:

Looking at the ruler on the left, which rods (A, B, C) do not fall directly on the 2 inch marker?

A and C

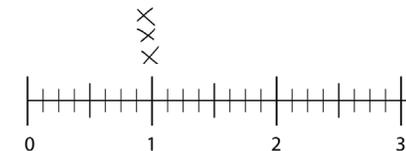
Measuring to the nearest inch is not the most precise way to measure these three rods. If you wanted a more precise measurement, what could you do?

Sample response:

round to a smaller unit, such as a half-inch

3. Students do a similar problem on their own.

Solution:



Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to measure and indicate rounded values on a line plot.
 (Targeted Error: Rounding to the wrong unit.)

1. Students study Ayden's work. Ayden has completed his work incorrectly.

2. Students respond to two questions related to Ayden's work:

- 1. Ayden mistakenly rounded to the nearest inch, rather than the nearest half-inch. What measurements should he have plotted for points A, B, and C?

A: 2 inches

B: 1.5 inches

C: 1 inch

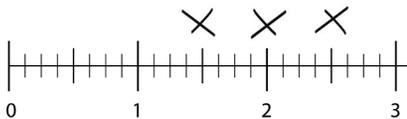
- 2. Which rod's position on the line plot needs to change? How do you know?

Sample response:

Rod B should change because it's closer to 1.5 inches than it is to 2 inches.

3. Students do a similar problem on their own.

Solution:



SET TWO

1. Study Ayden's incorrect work.

X

Name: Ayden

Using the ruler, measure each rod to the **nearest half-inch**. Complete the line plot using these measurements.

A

B

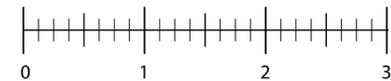
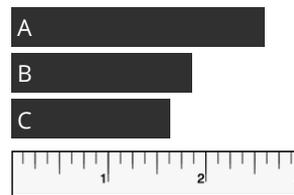
C

2. Answer these questions.

- 1. Ayden mistakenly rounded to the nearest inch, rather than the nearest half-inch. What measurements should he have plotted for points A, B, and C?
- 2. Which rod's position on the line plot needs to change? How do you know?

3. Then complete this one.

Using the ruler, measure each rod to the **nearest half-inch**. Complete the line plot using these measurements.



Name: _____ Date: _____

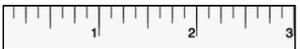
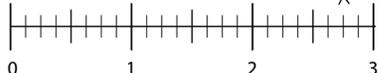
SET ONE

1. Study Parker's correct work.

 **Name:** PARKER

Using the ruler, measure each rod to the **nearest quarter-inch**. Complete the line plot using these measurements.

A 
B 
C 

2. Answer the question.

- Look at the line plot. Which of the measurements is the most frequent in this data set? Explain.

3. Then complete this one.

Using the ruler, measure each rod to the **nearest quarter-inch**. Complete the line plot using these measurements.

A 
B 
C 

Basic Operations with Fractions
Fraction Line Plots (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to determine frequency from a line plot.

- Students study Parker's work. Parker has completed his work correctly.**

- Students respond to a question related to Parker's work:**

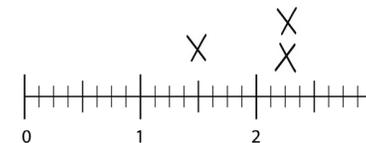
- Look at the line plot. Which of the measurements is the most frequent in this data set? Explain.

Sample response:

Two inches is the most frequent because two of the rods are about 2 inches long. Only one rod is about 2.75 inches long.

- Students do a similar problem on their own.**

Solution:



Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to measure and indicate rounded values on a line plot. (Targeted Error: Measuring the rod using only the "end point" of the rod, without looking to see where the rod began.)

1. Students study Maya's work. Maya has completed her work incorrectly.

2. Students respond to two questions related to Maya's work:

- Maya plotted the measurements just by looking at the end of each rod. Why is her answer incorrect?

Sample response:

The rods started at one inch, so she should have subtracted one from each of their end points before plotting.

- What measurements should she have plotted for points A, B, and C?

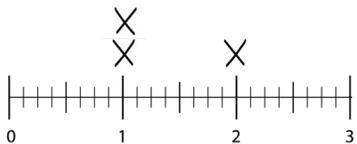
A: 2 inches

B: 1 inch

C: 2 inches

3. Students do a similar problem on their own.

Solution:



SET TWO

1. Study Maya's incorrect work.

X

Name: Maya

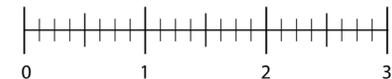
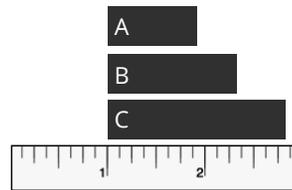
Using the ruler, measure each rod to the **nearest inch**. Complete the line plot using these measurements.

2. Answer these questions.

- Maya plotted the measurements just by looking at the end of each rod. Why is her answer incorrect?
- What measurements should she have plotted for points A, B, and C?

3. Then complete this one.

Using the ruler, measure each rod to the **nearest inch**. Complete the line plot using these measurements.



Name: _____ Date: _____

SET ONE

1. Study Katherine's correct work.

Name: Katherine

Write an expression to match the phrase. Then find the unknown.

One third the sum of 4 and 6

$\frac{1}{3} \times (4 + 6)$

$\frac{1}{3} \times 10$

$\frac{1}{3} \times 10$

$\frac{10}{3}$

3. Then complete this one.

Write an expression to match the phrase. Then find the unknown.

One fourth the sum of 7 and 2

2. Answer these questions.

- Circle the words in the phrase below that represent the whole.

One third the sum of 4 and 6

- How many parts is the whole being partitioned into?

- How many parts of the whole is Katherine trying to find?

Basic Operations with Fractions
Addition and Subtraction Expressions with Fractions

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to interpret the part/whole relationship when the expression is written in words.

1. Students study Katherine's work. Katherine has completed her work correctly.

2. Students respond to three questions related to Katherine's work:

- Circle the words in the phrase below that represent the whole.

One third the sum of 4 and 6

- How many parts is the whole being partitioned into?

3

- How many parts of the whole is Katherine trying to find?

1

3. Students do a similar problem on their own.

Expression:

$$\frac{1}{4} \times (7 + 2)$$

Solution:

$$\frac{9}{4}$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to interpret a written expression. (Targeted Error: Not including parentheses.)

1. Students study Jason's work. Jason has completed his work incorrectly.

2. Students respond to two questions related to Jason's work:

- Jason should have used parentheses when writing his expression. Why?

Sample response:

Without parentheses, Jason first multiplies $2 \times \frac{3}{4}$ then subtracts $\frac{1}{4}$. The expression is asking for you to subtract $\frac{1}{4}$ from $\frac{3}{4}$, then multiply by 2.

- Use the expression below to show where the parentheses should have been placed.

$$2 \times \left(\frac{3}{4} - \frac{1}{4} \right)$$

3. Students do a similar problem on their own.

Expression:

$$5 \times \left(\frac{2}{5} - \frac{1}{5} \right)$$

Solution:

$$\frac{5}{5}$$

SET TWO

1. Study Jason's incorrect work.

X Name: Jason

Write an expression to match the phrase. Then find the unknown.

Two times the difference between $\frac{3}{4}$ and $\frac{1}{4}$

$$2 \times \frac{3}{4} - \frac{1}{4}$$
$$\frac{6}{4} - \frac{1}{4}$$
$$\frac{5}{4}$$

2. Answer these questions.

- Jason should have used parentheses when writing his expression. Why?

- Use the expression below to show where the parentheses should have been placed.

$$2 \times \frac{3}{4} - \frac{1}{4}$$

3. Then complete this one.

Write an expression to match the phrase. Then find the unknown.

Five times the difference between $\frac{2}{5}$ and $\frac{1}{5}$

Name: _____ Date: _____

SET ONE

1. Study Kareem's correct work.

Name: Kareem

Write an expression to match the phrase. Then find the unknown.

Five times the quotient of 1 divided by 2

$$\frac{1}{2} \times 5 = \frac{5}{2}$$
$$5 \times \frac{1}{2} = \frac{5}{2}$$

2. Answer these questions.

- 1. Kareem could not decide which expression he wanted to use, so he wrote both. Why was it okay for him to begin one expression with $\frac{1}{2}$ and the other with 5?
- 2. Write another expression that also represents this phrase.

3. Then complete this one.

Write an expression to match the phrase. Then find the unknown.

Three times the quotient of 4 divided by 6

Basic Operations with Fractions
Multiplication and Division Expressions with Fractions

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that there are multiple ways to write a numerical expression based on the same written expression.

1. Students study Kareem's work. Kareem has completed his work correctly.

2. Students respond to two questions related to Kareem's work:

- 1. Kareem could not decide which expression he wanted to use, so he wrote both. Why was it okay for him to begin one expression with $\frac{1}{2}$ and the other with 5?

Sample response:

Multiplication is commutative. You can switch the order.

- 2. Write another expression that also represents this phrase.

Sample responses:

$$1 \div 2 \times 5$$

$$1 \times 5 \div 2$$

3. Students do a similar problem on their own.

Expression:

$$3 \times \frac{4}{6}$$

Solution:

$$2$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to interpret a written expression. (Targeted Error: Solving without considering order of operations.)

1. Students study Makayla's work. Makayla has completed her work incorrectly.

2. Students respond to two questions related to Makayla's work:

- In the step marked with an arrow, Makayla did not follow the order of operations. What operation should she have performed first?

Multiplication

- Do $3 + \frac{1}{2} \times 24$ and $3 + (\frac{1}{2} \times 24)$ mean the same thing? Explain.

Sample responses:

Yes, either way, the order of operations states that you must complete multiplication before addition.

Putting parentheses around the multiplication doesn't change the order since multiplication would be completed first anyway.

3. Students do a similar problem on their own.

Expression:

$$9 + \frac{2}{3} \times 15$$

Solution:

19

SET TWO

1. Study Makayla's incorrect work.

X Name: Makayla

Write an expression to match the phrase. Then find the unknown.

Add 3 to $\frac{1}{2}$ of 24

$3 + \frac{1}{2} \times 24$

$3\frac{1}{2} \times 24$

84

2. Answer these questions.

- In the step marked with an arrow, Makayla did not follow the order of operations. What operation should she have performed first?

- Do $3 + \frac{1}{2} \times 24$ and $3 + (\frac{1}{2} \times 24)$ mean the same thing? Explain.

3. Then complete this one.

Write an expression to match the phrase. Then find the unknown.

Add 9 to $\frac{2}{3}$ of 15

SET ONE

1. Study Jaylen's incorrect work.

X

Name: Jaylen

Solve.

$$\frac{1}{4} \times 20 = 1$$

3. Then complete this one.

Solve.

$$\frac{1}{3} \times 9 =$$

2. Answer these questions.

- 1. Jaylen correctly divided his diagram into 4 parts. Why did he do this?
- 2. Jaylen needed to find the value of each part. How could he have figured this out?
- 3. If the equation was $\frac{3}{4} \times 20$, how could Jaylen have determined the value of 3 of the 4 parts?

Multiplying and Dividing Fractions
Multiplying a Fraction by a Whole Number

Teacher Notes

SET 1 OF 2

TARGET:

Interpreting the product $(a/b) \times q$ as "a" parts of a partition of "q" into "b" equal parts. (Targeted Error: Representing the division problem but not knowing how to determine the value of each part of the partition.)

1. Students study Jaylen's work. Jaylen has completed his work incorrectly.

2. Students respond to three questions related to Jaylen's work:

- 1. Jaylen correctly divided his diagram into 4 parts. Why did he do this?

Sample response:

The denominator is four, so there are 4 parts.

- 2. Jaylen needed to find the value of each part. How could he have figured this out?

Sample response:

He should have divided 20 by 4. $20/4 = 5$.

- 3. If the equation was $\frac{3}{4} \times 20$, how could Jaylen have determined the value of 3 of the 4 parts?

Sample response:

He could have multiplied 5×3 .

3. Students do a similar problem on their own.

Solution:

3

Teacher Notes

SET 2 OF 2

TARGET:

Interpreting the product $(a/b) \times q$ as "a" parts of a partition of "q" into "b" equal parts.

1. Students study Jordan's work. Jordan has completed her work correctly.

2. Students respond to three questions related to Jordan's work:

- Why did Jordan divide 12 by 3?

Sample response:

She had to find the value of each part.

- Why did Jordan multiply 4 by 2?

Sample response:

She had to find the value of 2 of the parts.

- Use the numbers in the problem to make the following sentence represent the equation:

If 12 is partitioned into 3 equal parts, the value of 2 of those parts is 8.

3. Students do a similar problem on their own.

Solution:

9

SET TWO

1. Study Jordan's correct work.

✓ Name: Jordan

Solve.

$$\frac{2}{3} \times 12 = 8$$
$$12 \div 3 = 4$$
$$4 \times 2 = 8$$

A diagram shows a rectangle divided into 3 equal vertical parts. A bracket above the rectangle is labeled "12". A bracket below the rectangle is labeled "8". A stick figure stands to the right of the diagram.

2. Answer these questions.

- Why did Jordan divide 12 by 3?

- Why did Jordan multiply 4 by 2?

- Use the numbers in the problem to make the following sentence represent the equation:

If _____ is partitioned into _____ equal parts, the value of _____ of those parts is _____.

3. Then complete this one.

Solve.

$$\frac{3}{5} \times 15 =$$

Name: _____ Date: _____

SET ONE

1. Study Yesenia's incorrect work.

X Name: YESENIA

Solve.

$$\frac{2}{3} \times \frac{1}{4} = \frac{24}{12}$$
$$\frac{2}{3} \times \frac{1}{4}$$
$$\frac{2 \times 4}{3 \times 4} \times \frac{1 \times 3}{4 \times 3}$$
$$\frac{8}{12} \times \frac{3}{12}$$
$$\frac{24}{12}$$

2. Answer these questions.

- 1. In the step marked with an arrow, what was Yesenia's mistake? Explain.
- 2. Yesenia's strategy was to find common denominators. If she had correctly multiplied $\frac{8}{12} \times \frac{3}{12}$, would her strategy have worked? Explain your reasoning.

3. Then complete this one.

Solve.

$$\frac{6}{7} \times \frac{1}{2} =$$

Multiplying and Dividing Fractions

Multiplying a Fraction by a Fraction

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to multiply fractions. (Targeted Error: Applying the procedure of adding fractions—finding a common denominator and using that denominator in the answer—when multiplying fractions.)

1. Students study Yesenia's work. Yesenia has completed her work incorrectly.

2. Students respond to two questions related to Yesenia's work:

- 1. In the step marked with an arrow, what was Yesenia's mistake? Explain.

Sample response:

She didn't multiply 12 by 12. When multiplying fractions you multiply both the numerators together and the denominators together.

- 2. Yesenia's strategy was to find common denominators. If she had correctly multiplied $\frac{8}{12} \times \frac{3}{12}$, would her strategy have worked? Explain your reasoning.

Sample response:

Yes, $\frac{8}{12}$ and $\frac{3}{12}$ are still equivalent to $\frac{2}{3}$ and $\frac{1}{4}$.

3. Students do a similar problem on their own.

Solution:

$$\frac{6}{14} \text{ or } \frac{3}{7}$$

Teacher Notes

SET 2 OF 2

TARGET:

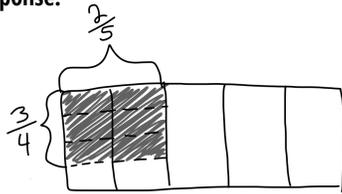
Understanding how to create a word problem to represent a number sentence.

1. Students study Peyton's work. Peyton has completed his work correctly.

2. Students respond to two questions related to Peyton's work:

- Draw a diagram to represent Peyton's word problem.

Sample response:



- Write a different word problem to represent $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$.

Sample response:

The farmer owns $\frac{2}{5}$ of an acre of land. She grows carrots on $\frac{3}{4}$ of that land. On how many acres does the farmer grow carrots?

3. Students do a similar problem on their own.

Solution:

$$\frac{2}{10} \text{ or } \frac{1}{5}$$

Word problems will vary.

The cookie recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar should the cook use to make $\frac{2}{5}$ of a batch of cookies?

SET TWO

1. Study Peyton's correct work.

Name: Peyton

Solve. Then write a word problem that represents the number sentence.

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$$

Word problem:

I had $\frac{2}{5}$ of a candy bar. But I only ate $\frac{3}{4}$ of that. How much of the candy bar did I eat?

2. Answer these questions.

- Draw a diagram to represent Peyton's word problem.

- Write a different word problem to represent $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$.

3. Then complete this one.

Solve. Then write a word problem that represents the number sentence.

$$\frac{2}{5} \times \frac{1}{2} =$$

Word problem:

Name: _____ Date: _____

SET ONE

1. Study Lydia's correct work.

Name: Lydia

Find the area of the rectangle below.

4 cm

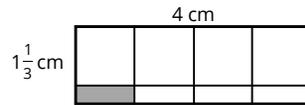
$$\begin{array}{cccc}
 | & | & | & | \\
 + & + & + & + \\
 \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
 \hline
 4 & + & \frac{3}{3} & + \frac{1}{3} \\
 & & 5 & + \frac{1}{3} \\
 & & 5\frac{1}{3} & \text{cm}^2
 \end{array}$$

2. Answer these questions.

- Lydia decided to break the rectangle into 8 smaller rectangles. How did she find the area of the rectangle shaded below?

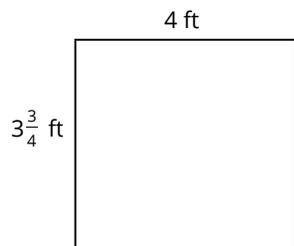


- How did she find the area of the smaller rectangle shaded below?



3. Then complete this one.

Find the area of the rectangle below.



Multiplying and Dividing Fractions

Area Problems (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how an area model breaks down $a*d$ e/f to $(a*d)+(a*e/f)$.

1. Students study Lydia's work. Lydia has completed her work correctly.

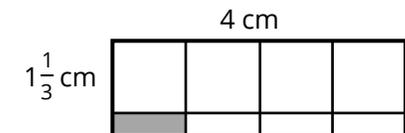
2. Students respond to two questions related to Lydia's work:

- Lydia decided to break the rectangle into 8 smaller rectangles. How did she find the area of the rectangle shaded below?



1 cm x 1 cm

- How did she find the area of the smaller rectangle shaded below?



1 cm x $\frac{1}{3}$ cm

3. Students do a similar problem on their own.

Solution:

15 ft^2

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to multiply whole numbers and fractions. (Targeted Error: Placing the same denominator of the fraction under the whole number when adding whole numbers and fractions.)

1. Students study Sebastian's work. Sebastian has completed his work incorrectly.

2. Students respond to two questions related to Sebastian's work:

- In the step marked with an arrow, Sebastian changed 10 to $\frac{10}{2}$. Do 10 and $\frac{10}{2}$ mean the same thing? Explain.

Sample response:

No, $\frac{10}{2}$ equals 5 because 10 divided by 2 is 5 .

- What fraction should Sebastian have changed 10 to? Explain.

Sample response:

$\frac{10}{1}$, $\frac{10}{1}$ means the same thing as 10 because 10 divided by 1 is 10 .

3. Students do a similar problem on their own.

Solution:

8 in^2

SET TWO

1. Study Sebastian's incorrect work.

X

Name: Sebastian

Find the area of the rectangle below.

5 cm

$2\frac{1}{2}$ cm

$$5 \times 2\frac{1}{2}$$

$$(5 \times 2) + (5 \times \frac{1}{2})$$

$$10 + \frac{5}{2}$$

$$\frac{10}{2} + \frac{5}{2}$$

$$\frac{15}{2} \text{ cm}^2$$

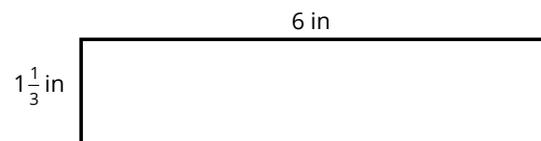
2. Answer these questions.

- In the step marked with an arrow, Sebastian changed 10 to $\frac{10}{2}$. Do 10 and $\frac{10}{2}$ mean the same thing? Explain.

- What fraction should Sebastian have changed 10 to? Explain.

3. Then complete this one.

Find the area of the rectangle below.



Teacher Notes

SET 2 OF 2

TARGET:

Understanding how an area model breaks down a $b/c * d e/f$ to $(a * d) + (a * e/f) + (b/c * d) + (b/c * e/f)$. (Targeted Error: Only finding 2 of 4 partial products.)

1. Students study Leah's work. Leah has completed her work incorrectly.

2. Students respond to two questions related to Leah's work:

- Complete the area model below to help Leah find the correct solution.

	4	$\frac{1}{3}$
2	8	$\frac{2}{3}$
$\frac{1}{3}$	$\frac{4}{3}$	$\frac{1}{9}$

- Based on your area model above, why should Leah have added together four multiplication expressions, rather than two?

Sample response:

She split each factor into two separate numbers. So, she needed to multiply each of the four numbers together.

3. Students do a similar problem on their own.

Solution:

$$18 \text{ mi}^2$$

SET TWO

1. Study Leah's incorrect work.

X

Name: LEAH

Find the area of the rectangle below.

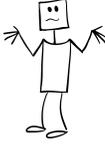
$4\frac{1}{3} \text{ km}$

 $2\frac{1}{3} \text{ km}$

$$(4 \times 2) + (\frac{1}{3} \times \frac{1}{3})$$

$$= 8 + \frac{1}{9}$$

$$= 8\frac{1}{9} \text{ km}^2$$



2. Answer these questions.

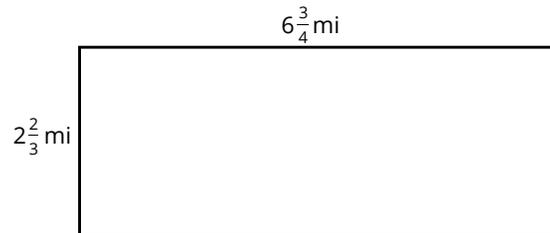
- Complete the area model below to help Leah find the correct solution.

	4	$\frac{1}{3}$
2		
$\frac{1}{3}$		

- Based on your area model above, why should Leah have added together four multiplication expressions, rather than two?

3. Then complete this one.

Find the area of the rectangle below.



Name: _____ Date: _____

SET ONE

1. Study Ryan's correct work.

Name: Ryan

Solve. Then draw a model to represent the number sentence.

$$\frac{1}{2} \times 8 = 4$$

$\frac{1}{2} \times 8$
 $\frac{1}{2} \times \frac{8}{1}$
 $\frac{8}{2}$
 (4)

8
4

$\frac{1}{2}$ of 8

2. Answer these questions.

⦿ A scale factor can change the size of the other factor. What is the scale factor in this problem?

⦿ Does multiplying by $\frac{1}{2}$ make the other factor bigger or smaller?

3. Then complete this one.

Solve. Then draw a model to represent the number sentence.

$$\frac{1}{3} \times 6 =$$

Multiplying and Dividing Fractions
Multiplication as Scaling (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Interpreting multiplication as scaling (resizing).

1. Students study Ryan's work. Ryan has completed his work correctly.

2. Students respond to two questions related to Ryan's work:

⦿ A scale factor can change the size of the other factor. What is the scale factor in this problem?

$$\frac{1}{2}$$

⦿ Does multiplying by $\frac{1}{2}$ make the other factor bigger or smaller?

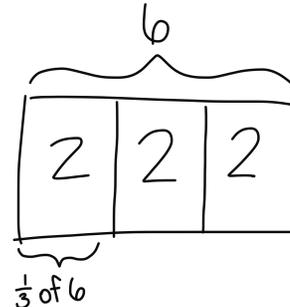
smaller

3. Students do a similar problem on their own.

Solution:

$$2$$

Sample model:



Teacher Notes

SET 2 OF 2

TARGET:

Understanding that $b/b \cdot a = a$. (Targeted Error: Believing $0/a \cdot a = a$.)

1. Students study Allison's work. Allison has completed her work incorrectly.
2. Students respond to three questions related to Allison's work:

- How could you show Allison that multiplying by $\frac{0}{10}$ **does** change the value of 10?

Sample response:

$0 \times 10 = 0$, so multiplying $\frac{0}{10} \times 10$ would be 0.

- What whole number do you need to multiply by to keep the value of 10? Explain.

Sample response:

You need to multiply by 1, since 1 multiplied by any number equals that number.

- Write a fraction that is equivalent to your number above.

Sample responses:

$\frac{3}{3}$; $\frac{4}{4}$; $\frac{5}{5}$

3. Students do a similar problem on their own.

Sample solution:

Any fraction equivalent to 1, except $\frac{1}{1}$.

Sample explanation:

I choose a fraction equal to 1 because 1×15 is 15.

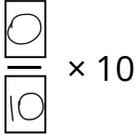
SET TWO

1. Study Allison's incorrect work.

X

Name: Allison

Determine a scale factor that **will not change** the value of 10. Explain your reasoning.



Explanation:

I choose $\frac{0}{10}$ because it will not change the value of 10.



2. Answer these questions.

- How could you show Allison that multiplying by $\frac{0}{10}$ **does** change the value of 10?
- What whole number do you need to multiply by to keep the value of 10? Explain.
- Write a fraction that is equivalent to your number above.

3. Then complete this one.

Determine a scale factor that **will not change** the value of 15. Explain your reasoning.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times 15$$

Explanation:

Name: _____ Date: _____

SET ONE

1. Study Jada's correct work.



Name: Jada

Determine a scale factor that **will decrease** the value of 27. Explain your reasoning.

$$\frac{\boxed{4}}{\boxed{5}} \times 27$$

Explanation:

$\frac{4}{5}$ is less than 1 so it will decrease the value of 27



2. Answer these questions.

- 1. Why did Jada use a numerator that was less than the denominator?
- 2. Is there another fraction Jada could have chosen to decrease the value of 27? Explain.

3. Then complete this one.

Determine a scale factor that **will decrease** the value of 31. Explain your reasoning.

$$\frac{\boxed{}}{\boxed{}} \times 31$$

Explanation:

Multiplying and Dividing Fractions
Multiplication as Scaling (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that, when multiplying a number by an unknown to get an answer that is less than your original number, you must multiply by a number less than 1.

1. Students study Jada's work. Jada has completed her work correctly.

2. Students respond to two questions related to Jada's work:

- 1. Why did Jada use a numerator that was less than the denominator?

Sample response:

She needed a fraction less than one so the answer would be less than 27.

- 2. Is there another fraction Jada could have chosen to decrease the value of 27? Explain.

Sample response:

Yes, she could have chosen any fraction less than 1.

3. Students do a similar problem on their own.

Sample solution:

Any number less than 1.

Sample explanation:

[Number less than 1] is less than 1 so it will decrease the value of 31.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding that multiplying a number by an unknown to get an answer that is greater than your original number, you must multiply by a number greater than 1. (Targeted Error: Thinking that multiplying by any number will increase the value of the original number.)

1. Students study Nathan's work. Nathan has completed his work incorrectly.

2. Students respond to two questions related to Nathan's work:

- $\frac{2}{3}$ is less than 1. Will multiplying by a number less than one increase or decrease the value of 12?

decrease

- What fraction could Nathan multiply by in order to increase the value of 12? Fill in the boxes below.

$$\frac{\square}{\square} \times 12$$

any number greater than 1

3. Students do a similar problem on their own.

Solution:

any number greater than 1

Sample explanation:

Multiplying by a number greater than 1 will increase the value of 5.

SET TWO

1. Study Nathan's incorrect work.

X

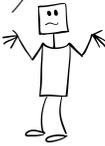
Name: Nathan

Determine a scale factor that **will increase** the value of 12. Explain your reasoning.

$$\frac{2}{3} \times 12$$

Explanation:

Multiplying by any number will increase the value of 12.



2. Answer these questions.

- $\frac{2}{3}$ is less than 1. Will multiplying by a number less than one increase or decrease the value of 12?

- What fraction could Nathan multiply by in order to increase the value of 12? Fill in the boxes below.

$$\frac{\square}{\square} \times 12$$

3. Then complete this one.

Determine a scale factor that **will increase** the value of 5. Explain your reasoning.

$$\frac{\square}{\square} \times 5$$

Explanation:

Name: _____ Date: _____

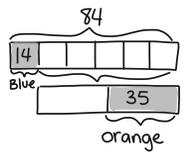
SET ONE

1. Study Lila's correct work.

 Solve. Write your answer in sentence form.

Name: Lila

A 1st grader is sorting his colored blocks. He has a total of 84 blocks. $\frac{1}{6}$ of the blocks are blue, and $\frac{1}{2}$ of the remaining blocks are orange. How many blocks are orange?



Blue = $\frac{1}{6}$ of 84 = $\frac{1}{6} \times \frac{84}{1} = \frac{84}{6} = 14$

Orange = $\frac{1}{2}$ of 70 = $\frac{1}{2} \times \frac{70}{1} = \frac{70}{2} = 35$

He has 35 orange blocks.



2. Answer the question.

- In the step marked with an arrow, why did Lila multiply $\frac{1}{2}$ by 70 instead of 84?

3. Then complete this one.

Solve. Write your answer in sentence form.

An art teacher is buying tiles to make a mosaic design. The design has 72 tiles in total. $\frac{1}{8}$ of the tiles are red, and $\frac{1}{3}$ of the remaining tiles are purple. How many purple tiles should she purchase?

Multiplying and Dividing Fractions

Multiplication Word Problems

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to interpret a multi-step word problem.

1. Students study Lila's work. Lila has completed her work correctly.

2. Students respond to a question related to Lila's work:

- In the step marked with an arrow, why did Lila multiply $\frac{1}{2}$ by 70 instead of 84?

Sample response:

$\frac{1}{2}$ of the remaining blocks are orange, not $\frac{1}{2}$ of all 84 blocks.

3. Students do a similar problem on their own.

Solution:

She needs to buy 21 purple tiles.

Teacher Notes

SET 2 OF 2

TARGET:

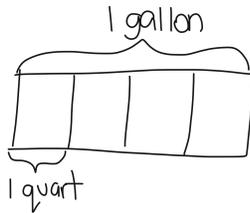
Understanding how to use conversion factors to solve. (Targeted Error: Using the wrong conversion factor.)

1. Students study Brady's work. Brady has completed his work incorrectly.

2. Students respond to two questions related to Brady's work:

- Draw a picture to show that 1 gallon is equivalent to 4 quarts.

Sample response:



- Why should Brady have multiplied by $\frac{1}{4}$ gallon, rather than 4 gallons?

Sample responses:

4 quarts = 1 gallon.

1 quart is $\frac{1}{4}$ of a gallon.

The answer needs to be less than 64 since it takes 4 quarts to make up 1 gallon.

3. Students do a similar problem on their own.

Solution:

The cook made 14 gallons of ice cream.

SET TWO

1. Study Brady's incorrect work.

X

Name: Brady

Solve. Write your answer in sentence form.

The farmer has 64 quarts of milk. How many gallons of milk does the farmer have? Hint: 1 gallon is equivalent to 4 quarts.

$$64qt = 64 \times 4gal$$
$$= 257gal$$

The farmer has 257 gallons of milk.

2. Answer these questions.

- Draw a picture to show that 1 gallon is equivalent to 4 quarts.
- Why should Brady have multiplied by $\frac{1}{4}$ gallon, rather than 4 gallons?

3. Then complete this one.

Solve. Write your answer in sentence form.

A cook made 56 quarts of ice cream. How many gallons of ice cream did the cook make? Hint: 1 gallon is equivalent to 4 quarts.

Name: _____ Date: _____

SET ONE

1. Study Callie's correct work.

Name: Callie

Solve. Write your answer in sentence form.

A carpenter is building a square table. The length of one side is $\frac{3}{5}$ of a meter. What is the total area of the table?

$\frac{3}{5}m$ $\frac{3}{5}m$

$\frac{3}{5}m \times \frac{3}{5}m$

$\frac{3 \times 3}{5 \times 5} m^2$

$\frac{9}{25} m^2$

The area of the table is $\frac{9}{25} m^2$.

2. Answer the question.

- How did Callie know to multiply $\frac{3}{5}$ by $\frac{3}{5}$?

3. Then complete this one.

Solve. Write your answer in sentence form.

A farmer is planting a square flower garden that is $\frac{5}{6}$ of a yard on one side. What is the total area of his garden?

Multiplying and Dividing Fractions
Area Word Problems with Fractions

Teacher Notes

SET 1 OF 2

TARGET:

Interpreting a word problem and determining how to solve for the area of a square.

1. Students study Callie's work. Callie has completed her work correctly.

2. Students respond to a question related to Callie's work:

- How did Callie know to multiply $\frac{3}{5}$ by $\frac{3}{5}$?

Sample response:

The table is a square, and the sides of a square are all equal. So, if one side is $\frac{3}{5}$ of a meter, all sides would be $\frac{3}{5}$ of a meter.

3. Students do a similar problem on their own.

Solution:

The total area of the garden is $\frac{25}{36} yd^2$.

Teacher Notes

SET 2 OF 2

TARGET:

Interpreting a word problem and determining how to solve for the area of a square. (Targeted Error: Adding sides to find perimeter, rather than multiplying to find the area.)

1. Students study Elliot's work. Elliot has completed his work incorrectly.

2. Students respond to two questions related to Elliot's work:

- Did Elliot solve for the perimeter or the area of the carpet? How can you tell?

Sample response:

He solved for perimeter because he added the length of all the sides together.

- Based on the word problem, should Elliot have solved for the perimeter or the area of the carpet? How do you know?

Sample response:

He should have solved for area because he was supposed to figure out how much carpet was needed to fill a space.

3. Students do a similar problem on their own.

Solution:

The area of the wall is $55\frac{1}{4}$ ft².

SET TWO

1. Study Elliot's incorrect work.

X

Name: Elliot

Solve. Write your answer in sentence form.

A woman needs to buy new carpet for her bedroom. The room measures $9\frac{1}{4}$ ft by $8\frac{1}{4}$ ft. How much carpet should she buy?



$$9\frac{1}{4} + 8\frac{1}{4} + 9\frac{1}{4} + 8\frac{1}{4}$$

$$9 + 8 + 9 + 8 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$34 + \frac{4}{4}$$

$$34 + 1$$

$$35$$

She needs to buy 35 feet of carpet.



2. Answer these questions.

- Did Elliot solve for the perimeter or the area of the carpet? How can you tell?
- Based on the word problem, should Elliot have solved for the perimeter or the area of the carpet? How do you know?

3. Then complete this one.

Solve. Write your answer in sentence form.

A man wants to paint one wall in his office red. The wall is $6\frac{1}{2}$ ft by $8\frac{1}{2}$ ft. What is the area of the wall he wants to paint?

Name: _____ Date: _____

SET ONE

1. Study Aubrey's correct work.

✓ Name: Aubrey

Solve.

$$\frac{1}{6} \div 2 = \frac{1}{12}$$

$\frac{1}{6} \div 2 = \frac{1}{12}$

2. Answer these questions.

- Why did Aubrey divide the top diagram into six parts?
- Why did Aubrey divide the bottom diagram into twelve parts?

3. Then complete this one.

Solve.

$$\frac{1}{4} \div 3 =$$

Multiplying and Dividing Fractions
Division of a Fraction by Whole Number

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to represent dividing fractions using a tape diagram.

1. Students study Aubrey's work. Aubrey has completed her work correctly.

2. Students respond to two questions related to Aubrey's work:

- Why did Aubrey divide the top diagram into six parts?

Sample responses:

The denominator is 6 so she was showing what $\frac{1}{6}$ looks like by creating 6 sections.

Her fraction was $\frac{1}{6}$, so she needed 1 of 6 equal parts.

- Why did Aubrey divide the bottom diagram into twelve parts?

Sample response:

She was dividing each $\frac{1}{6}$ section by 2, which created 12 sections.

3. Students do a similar problem on their own.

Solution:

$$\frac{1}{12}$$

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to divide a fraction by a whole number. (Targeted Error: Dividing the fraction by $1/a$, rather than $a/1$.)

1. Students study Zach's work. Zach has completed his work incorrectly.
2. Students respond to three questions related to Zach's work:

- 1. Rather than dividing by 4, what number did Zach divide $\frac{1}{8}$ by?

$$\frac{1}{4}$$

- 2. Partition the diagram to represent $\frac{1}{8} \div 4$.

Sample response:



- 3. Based on the diagram above, should the answer be smaller or larger than $\frac{1}{8}$?

smaller

3. Students do a similar problem on their own.

Solution:

$$\frac{1}{15}$$

SET TWO

1. Study Zach's incorrect work.

X

Name: ZACH

Solve.

$$\frac{1}{8} \div 4 = \frac{1}{2}$$

$$\frac{1}{8} \div 4$$

$$\frac{1}{8 \div 4}$$

$$\frac{1}{2}$$

2. Answer these questions.

- 1. Rather than dividing by 4, what number did Zach divide $\frac{1}{8}$ by?

- 2. Partition the diagram to represent $\frac{1}{8} \div 4$.



- 3. Based on the diagram above, should the answer be smaller or larger than $\frac{1}{8}$?

3. Then complete this one.

Solve.

$$\frac{1}{5} \div 3 =$$

Name: _____ Date: _____

SET ONE

1. Study Grayson's correct work.

Name: GRAYSON

Solve.

$$5 \div \frac{1}{2} = 10$$

0 1 2 3 4 5
← $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$ $\frac{7}{2}$ $\frac{8}{2}$ $\frac{9}{2}$ $\frac{10}{2}$ →



3. Then complete this one.

Solve.

$$8 \div \frac{1}{3} =$$

2. Answer these questions.

- When you divide a whole number by a whole number, is the quotient always bigger or smaller than the dividend?
- Why is Grayson's quotient bigger than the dividend?

Multiplying and Dividing Fractions
Division of a Whole Number by Fraction

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that division does not always lead to a smaller answer.

1. Students study Grayson's work. Grayson has completed his work correctly.

2. Students respond to two questions related to Grayson's work:

- When you divide a whole number by a whole number, is the quotient always bigger or smaller than the dividend?

smaller

- Why is Grayson's quotient bigger than the dividend?

Sample response:

When you divide by a fraction less than 1, you are breaking the whole number into smaller parts, so there are more parts.

3. Students do a similar problem on their own.

Solution:

24

Teacher Notes

SET 2 OF 2

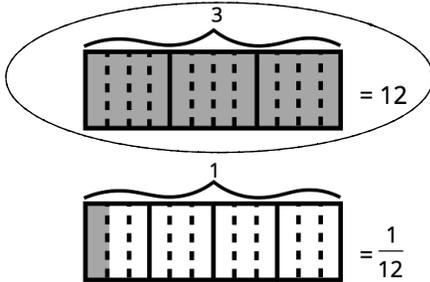
TARGET:

Understanding how to match a word problem to a diagram representing a numerical expression. (Targeted Error: Misinterpreting the word problem.)

1. Students study Lucy's work. Lucy has completed her work incorrectly.

2. Students respond to two questions related to Lucy's work:

- Circle the diagram that represents $3 \div \frac{1}{4}$.



- Use the diagrams above to explain why the word problem Lucy circled does not represent $3 \div \frac{1}{4}$.

Sample response:

The first word problem starts with $\frac{1}{4}$ and divides it by 3. We need to start with 3 and divide it by $\frac{1}{4}$.

3. Students do a similar problem on their own.

Solution:

A class is having blueberries for a snack. There are 5 cups of blueberries. If each student has $\frac{1}{5}$ of a cup of blueberries, how many students will get a snack?

Mark has $\frac{1}{5}$ of a bag of M&Ms. If he splits them evenly among 5 friends, how much of the bag will each friend get?

SET TWO

1. Study Lucy's incorrect work.

X

Name: Lucy

Circle the word problem that correctly represents the expression below.

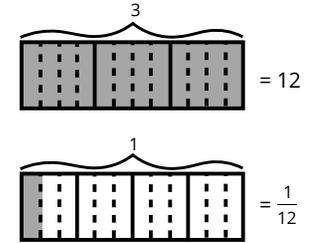
$3 \div \frac{1}{4}$

A pitcher is filled with $\frac{1}{4}$ of a gallon of lemonade. If it is split evenly into 3 glasses, how much will be in each glass?

Jamie is painting her house. Each wall takes $\frac{1}{4}$ of a gallon of paint. If Jamie has 3 gallons of paint, how many walls can she paint?

2. Answer these questions.

- Circle the diagram that represents $3 \div \frac{1}{4}$.



- Use the diagrams above to explain why the word problem Lucy circled does not represent $3 \div \frac{1}{4}$.

3. Then complete this one.

Circle the word problem that correctly represents the expression below.

$$5 \div \frac{1}{5}$$

A class is having blueberries for a snack. There are 5 cups of blueberries. If each student has $\frac{1}{5}$ of a cup of blueberries, how many students will get a snack?

Mark has $\frac{1}{5}$ of a bag of M&Ms. If he splits them evenly among 5 friends, how much of the bag will each friend get?

Name: _____ Date: _____

SET ONE

1. Study Jose's correct work.

 Name: Jose

Solve. Write your answer in sentence form.

A builder has a piece of rope that is $\frac{1}{3}$ m long. She needs to cut it into 6 equal pieces. How long will each piece be?

$\frac{1}{3} \div 6$
1 third $\div 6$
6 eighths $\div 6$
1 eighteenth
Each piece of rope will be $\frac{1}{18}$ m long.



3. Then complete this one.

Solve. Write your answer in sentence form.

A bag of pebbles weighs $\frac{1}{2}$ of a pound. A child is splitting this bag into 3 equal piles. How much will each pile weigh?

2. Answer these questions.

- Look at the step marked with an arrow. Partition the model to prove that 1 third is equivalent to 6 eighths.



- Why does changing 1 third to 6 eighths make it easier to use mental math to solve the problem?

Multiplying and Dividing Fractions
Division Word Problems (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to use a diagram and mental math to solve a division word problem.

1. Students study Jose's work. Jose has completed his work correctly.

2. Students respond to two questions related to Jose's work:

- Look at the step marked with an arrow. Partition the model to prove that 1 third is equivalent to 6 eighths.

Sample response:



- Why does changing 1 third to 6 eighths make it easier to use mental math to solve the problem?

Sample responses:

It gave him a value that could easily be divided by the divisor of 6.

$\frac{6}{18}$ divided by $\frac{6}{1} = \frac{1}{18}$. The numerators were the same.

3. Students do a similar problem on their own.

Solution:

Each pile weighs $\frac{1}{6}$ of a pound.

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to write a division expression based on a word problem.
(Targeted Error: Switching the dividend and divisor.)

1. Students study Mariah's work. Mariah has completed her work incorrectly.

2. Students respond to two questions related to Mariah's work:

- Based on the word problem, is it reasonable that the cook used 16 cups of pepperoni on each pizza? Explain.

Sample response:

No, the cook only had $\frac{1}{4}$ cup pepperoni to start with.

- Mariah set up her expression incorrectly. Why doesn't it make sense to divide 4 by $\frac{1}{4}$?

Sample responses:

She is sharing $\frac{1}{4}$ cup among 4 pizzas, NOT having 4 cups on only $\frac{1}{4}$ of a pizza.

She only had $\frac{1}{4}$ cup of pepperoni to split between the 4 pizzas. So the $\frac{1}{4}$ had to be divided by 4.

3. Students do a similar problem on their own.

Solution:

He used $\frac{1}{21}$ of a gallon in each bowl.

SET TWO

1. Study Mariah's incorrect work.

X Name: Mariah

Solve. Write your answer in sentence form.

The cook used $\frac{1}{4}$ of a cup of pepperoni to make 4 pizzas. She used the same amount of pepperoni on each pizza. How many cups of pepperoni did she use on each pizza?

$4 \div \frac{1}{4}$
 4×4
16

She used 16 cups of pepperoni on each pizza.

2. Answer these questions.

- Based on the word problem, is it reasonable that the cook used 16 cups of pepperoni on each pizza? Explain.
- Mariah set up her expression incorrectly. Why doesn't it make sense to divide 4 by $\frac{1}{4}$?

3. Then complete this one.

Solve. Write your answer in sentence form.

The baker used $\frac{1}{3}$ of a gallon of milk to make 7 bowls of pudding. He used the same amount of milk in each bowl. How much milk did he use in each bowl of pudding?

Name: _____ Date: _____

SET ONE

1. Study Alana's correct work.

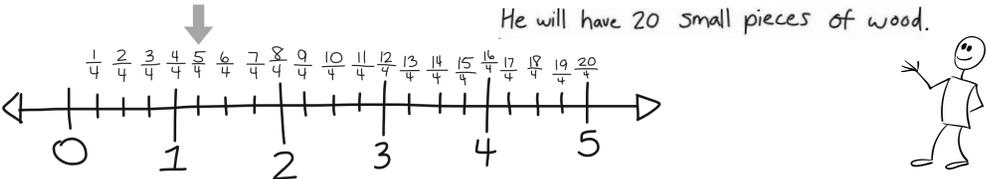


Name: Alana

Draw a picture to represent the problem. Then solve. Write your answer in sentence form.

A carpenter has 5 pieces of wood. If he splits each piece of wood into fourths, how many smaller pieces of wood will he have?

He will have 20 small pieces of wood.



2. Answer the question.

- In the place marked with an arrow, Alana continued marking the number line with $\frac{5}{4}$ instead of starting again with $\frac{1}{4}$. Why was this helpful?

3. Then complete this one.

Draw a picture to represent the problem. Then solve. Write your answer in sentence form.

The cook made 4 pans of lasagna. If he splits each pan into fifths, how many total pieces of lasagna will he have?

**Multiplying and Dividing Fractions
 Division Word Problems (Part 2)**

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to use a number line when dividing fractions.

- Students study Alana's work. Alana has completed her work correctly.**

- Students respond to a question related to Alana's work:**

- In the place marked with an arrow, Alana continued marking the number line with $\frac{5}{4}$ instead of starting again with $\frac{1}{4}$. Why was this helpful?

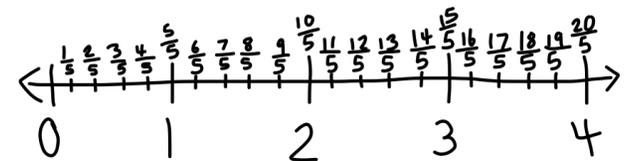
Sample response:

She was calculating the number of fourths that are in 5 wholes.

- Students do a similar problem on their own.**

Sample solution:

The cook will have 20 pieces of lasagna.



Teacher Notes

SET 2 OF 2

TARGET:

Understanding that a divided by b/c does not equal $b/(a$ divided by $c)$.
 (Targeted Error: Converting a whole number to $1/a$, rather than $a/1$.)

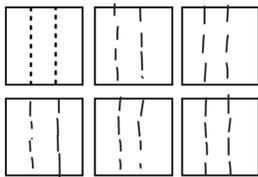
1. Students study Greg's work. Greg has completed his work incorrectly.
2. Students respond to three questions related to Greg's work:

- Greg made a mistake in the step marked with an arrow. Explain why $6 \div \frac{1}{3}$ is not the same as $\frac{1}{6+3}$.

Sample response:

6 is actually $\frac{6}{1}$ not $\frac{1}{6}$. Greg was supposed to divide 6 by the numerator and 1 by the denominator.

- Finish partitioning each square of fabric below into thirds.



- Using the diagram above, should Greg's answer be less than or greater than 6? Explain.

Sample response:

It should be greater than 6 because each of the 6 squares of fabric are split into multiple pieces.

3. Students do a similar problem on their own.

Solution:

He can make 28 pieces (fourths).

SET TWO

1. Study Greg's incorrect work.

X

Name: Greg

Solve. Write your answer in sentence form.

To make a quilt, the seamstress is dividing squares of fabric into thirds. How many pieces can she make from 6 squares of fabric?

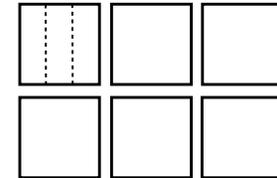
$6 \div \frac{1}{3}$
 $\frac{1}{6+3}$
 $\frac{1}{2}$

She can make $\frac{1}{2}$ of a piece.

2. Answer these questions.

- Greg made a mistake in the step marked with an arrow. Explain why $6 \div \frac{1}{3}$ is not the same as $\frac{1}{6+3}$.

- Finish partitioning each square of fabric below into thirds.



- Using the diagram above, should Greg's answer be less than or greater than 6? Explain.

3. Then complete this one.

Solve. Write your answer in sentence form.

For a craft project, the teacher is dividing paper plates into fourths. How many pieces can he make from 7 paper plates?

Name: _____ Date: _____

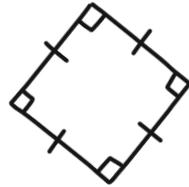
SET ONE

1. Study Sarah's correct work.

 Name: Sarah

Draw the figure described. List two additional names for the figure described.

A rhombus with four 90° angles



1. Quadrilateral
2. Rectangle



2. Answer these questions.

- 1. Why can Sarah's figure be called a quadrilateral?
- 2. Why can Sarah's figure be called a rectangle?
- 3. Why can Sarah's figure be called by multiple names?

3. Then complete this one.

Draw the figure described. List two additional names for the figure described.

- A trapezoid with exactly two 90° angles
- 1.
 - 2.

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that some figures can have more than one name.

1. Students study Sarah's work. Sarah has completed her work correctly.

2. Students respond to three questions related to Sarah's work:

- 1. Why can Sarah's figure be called a quadrilateral?

Sample response:

It has 4 sides, and quadrilaterals have 4 sides.

- 2. Why can Sarah's figure be called a rectangle?

Sample response:

It has two sets of parallel sides and all right angles.

- 3. Why can Sarah's figure be called by multiple names?

Sample response:

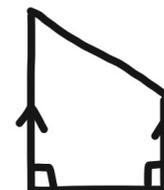
It has the properties of both shapes.

3. Students do a similar problem on their own.

Sample solution:

1. quadrilateral

2. polygon



(Drawings must have: four sides, only one set of parallel sides, exactly two 90° angles.)

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the attributes of quadrilaterals. (Targeted Error: Listing incorrect attributes for a rhombus.)

1. Students study Aaron's work. Aaron has completed his work incorrectly.

2. Students respond to two questions related to Aaron's work:

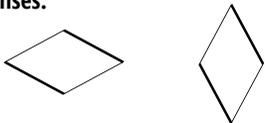
- ⦿ Aaron's second attribute is not correct for the figure described. Explain why a rhombus does **not** always have to have four equal angles.

Sample response:

A rhombus just needs opposite equal angles.

- ⦿ Draw a rhombus that does not have four equal angles.

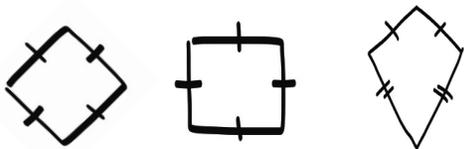
Sample responses:



(Drawings must have: four equal length sides, opposite parallel sides, opposite equal angles, and diagonals that are perpendicular and bisect each other.)

3. Students do a similar problem on their own.

Sample solutions:



(Drawings must have: two sets of equal adjacent sides; four sides; perpendicular diagonals; congruent opposite angles.)

SET TWO

1. Study Aaron's incorrect work.

X

Name: Aaron

Draw the figure described. List two attributes for the figure described.

A parallelogram that is also a rhombus

- Opposite sides are parallel
- All angles are always equal

2. Answer these questions.

- ⦿ Aaron's second attribute is not correct for the figure described. Explain why a rhombus does **not** always have to have four equal angles.

- ⦿ Draw a rhombus that does not have four equal angles.

3. Then complete this one.

Draw the figure described. List two attributes for the figure described.

A quadrilateral that is also a kite

1.

2.

Name: _____ Date: _____

SET ONE

1. Study Eli's correct work.

✓ Name: Eli

Check all that apply.

A square is **ALWAYS** a _____.

- Parallelogram
- Polygon
- Trapezoid
- Rectangle
- Rhombus
- Quadrilateral



2. Answer these questions.

☞ Why is a square always a rectangle?

☞ Is a rectangle always a square? Explain.

3. Then complete this one.

Check all that apply.

A parallelogram is **ALWAYS** a _____.

- Square
- Polygon
- Trapezoid
- Rectangle
- Rhombus
- Quadrilateral

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that a square is always a rectangle, but a rectangle is not always a square.

1. Students study Eli's work. Eli has completed his work correctly.

2. Students respond to two questions related to Eli's work:

☞ Why is a square always a rectangle?

Sample response:

Rectangles do not have any attributes that are not the attributes of a square. They both have four 90 degree angles, opposite equal sides, and two sets of parallel sides.

☞ Is a rectangle always a square? Explain.

Sample response:

No, a rectangle only has to have 2 pairs of opposite equal sides. The sides of a square must all be of sides of equal length.

3. Students do a similar problem on their own.

Solution:

Polygon

Quadrilateral

Teacher Notes

SET 2 OF 2

TARGET:

Understanding the attributes of a quadrilateral. (Targeted Error: Misidentifying a 5-sided figure as a quadrilateral.)

1. Students study Nora's work. Nora has completed her work incorrectly.

2. Students respond to two questions related to Nora's work:

- Nora chose figure b because it has a set of parallel sides. What about figure b does not fit the description?

Sample responses:

Figure b has 5 sides.

A quadrilateral must have 4 sides.

- Which figure represents a quadrilateral that has one set of parallel sides?

figure c

3. Students do a similar problem on their own.

Solution:

figure d

SET TWO

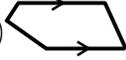
1. Study Nora's incorrect work.

X Name: Nora

Select the figure that best represents the description.

A quadrilateral that has one set of parallel sides.

a. 

b. 

c. 

d. 



2. Answer these questions.

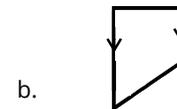
- Nora chose figure b because it has a set of parallel sides. What about figure b does not fit the description?

- Which figure represents a quadrilateral that has one set of parallel sides?

3. Then complete this one.

Select the figure that best represents the description.

A quadrilateral that is also a parallelogram.



Name: _____ Date: _____

SET ONE

1. Study Julian's correct work.

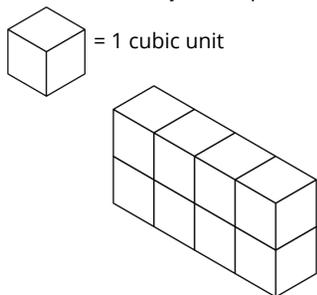
A handwritten student work page for Julian. At the top left is a large checkmark. To its right, the name "Julian" is written in cursive. Below the name, the text "Find the volume of the shape." is written. To the left of the main shape, a single cube is drawn and labeled "= 1 cubic unit". The main shape is a 3x2x2 rectangular prism, drawn as a 2x2 grid of cubes with a third cube on top of the front-left cube. Below the shape, the text "6 cubic units" is written. To the right of the shape is a simple stick figure drawing of a person with arms raised.

2. Answer the question.

- How can you tell that the volume of the shape is 6 cubic units?

3. Then complete this one.

Find the volume of the shape.



Teacher Notes

SET 1 OF 2

TARGET:

Understanding that a shape made of "n" unit cubes has the volume of "n" cubic units.

1. Students study Julian's work. Julian has completed his work correctly.

2. Students respond to a question related to Julian's work:

- How can you tell that the volume of the shape is 6 cubic units?

Sample response:

The shape is made up of 6 unit cubes.

3. Students do a similar problem on their own.

Solution:

8 cubic units

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to differentiate shapes with more or less volume.
(Targeted Error: Identifying a "taller" shape as having more volume without counting the unit cubes.)

1. Students study Skylar's work. Skylar has completed her work incorrectly.

2. Students respond to two questions related to Skylar's work:

- Skylar thought that the taller shape had a greater volume. Explain why this is not always correct.

Sample responses:

Volume is based on more than one dimension.

Volume is the amount of 3-dimensional space something takes up, not how tall it is.

- What are the volumes of Shape 1 and Shape 2? Explain your reasoning.

Sample response:

They both have a volume of 3 cubic units because they are both made up of 3 unit cubes.

3. Students do a similar problem on their own.

Solution:

Shapes 1 and 2 have the same volume.

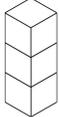
SET TWO

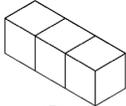
1. Study Skylar's incorrect work.

X Name: Skylar

Choose the statement that most accurately compares the volumes of the shapes below.

 = 1 cubic unit


1


2

Shape 1 has greater volume than Shape 2.

Shape 2 has greater volume than Shape 1.

Shapes 1 and 2 have the same volume.



2. Answer these questions.

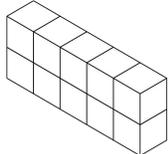
- Skylar thought that the taller shape had a greater volume. Explain why this is not always correct.

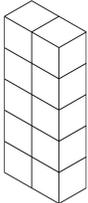
- What are the volumes of Shape 1 and Shape 2? Explain your reasoning.

3. Then complete this one.

Choose the statement that most accurately compares the volumes of the shapes below.

 = 1 cubic unit


1


2

Shape 1 has greater volume than Shape 2.

Shape 2 has greater volume than Shape 1.

Shapes 1 and 2 have the same volume.

Name: _____ Date: _____

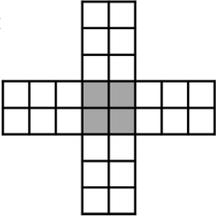
SET ONE

1. Study Isaiah's incorrect work.

X Name: Isaiah

Imagine this net were folded into a box. Find the volume of the box. Explain your reasoning.

 = 1 cubic unit



Volume: 4 cubic units

Explanation:
I know at least 4 unit cubes will fit in the box because of how big it is.



2. Answer these questions.

- Isaiah thought volume was any number of cubes that could fit in a box. Explain why his thinking is incorrect.

- What is the actual volume of the box? Explain how you know.

Geometry/Volume
Properties of Volume (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

*Understanding how to find the volume of a net that is folded into a box.
(Targeted Error: Misinterpreting the definition of volume.)*

1. Students study Isaiah's work. Isaiah has completed his work incorrectly.

2. Students respond to two questions related to Isaiah's work:

- Isaiah thought volume was any number of cubes that could fit in a box. Explain why his thinking is incorrect.

Sample response:

The volume is the number of cubes that fill the box completely.

- What is the actual volume of the box? Explain how you know.

Sample response:

The base layer has 4 cubes, and there are three layers, so the volume is 12 cubic units.

3. Students do a similar problem on their own.

Solution:

6 cubic units

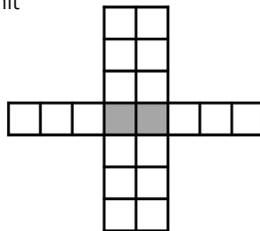
Sample explanation:

The bottom layer has 2 cubes, and there are 3 layers.

3. Then complete this one.

Imagine this net were folded into a box. Find the volume of the box. Explain your reasoning.

 = 1 cubic unit



Volume: _____

Explanation: _____

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to determine the volume of a net that is folded into a box, based on the volume of another box.

1. Students study Riley's work. Riley has completed her work correctly.

2. Students respond to three questions related to Riley's work:

How many unit cubes are in the base of each box?

A 4 B 8

What is the height of each box?

A 2 B 2

Based on your answers to the left, how did Riley know that the volume of Box B was twice as much as the volume of Box A?

Sample response:

The base was twice as much, and the height was the same.

3. Students do a similar problem on their own.

Solution:

24 cubic units

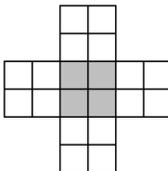
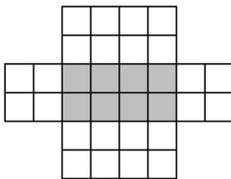
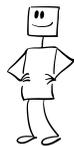
SET TWO

1. Study Riley's correct work.



Imagine these nets were folded into boxes. Use Box A to help you find the volume of Box B. Name: Riley

A: Volume: 8 cubic units B: Volume: 16 cubic units  = 1 cubic unit

2. Answer these questions.

How many unit cubes are in the base of each box?

A _____ B _____

What is the height of each box?

A _____ B _____

Based on your answers to the left, how did Riley know that the volume of Box B was twice as much as the volume of Box A?

3. Then complete this one.

Imagine these nets were folded into boxes. Use Box A to help you find the volume of Box B.

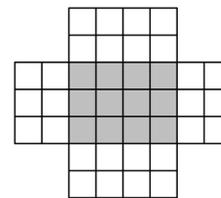
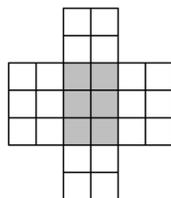
A:

Volume: 12 cubic units

B:

Volume: _____

 = 1 cubic unit



Name: _____ Date: _____

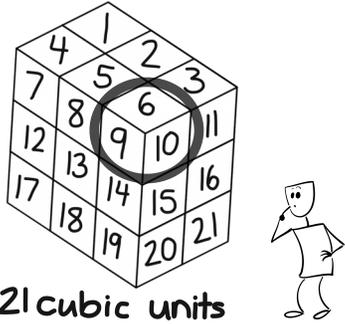
SET ONE

1. Study Santiago's incorrect work.

X Name: Santiago

Find the volume of the rectangular prism by counting the number of cubes.

 = 1 cubic unit



21 cubic units

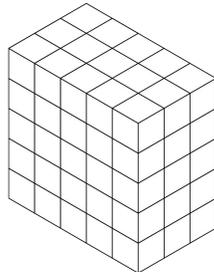
2. Answer the question.

- How can you tell from the circled cube that Santiago didn't count the number of cubes correctly?

3. Then complete this one.

Find the volume of the rectangular prism by counting the number of cubes.

 = 1 cubic unit



Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to find the volume of a rectangular prism by counting unit cubes. (Targeted Error: Counting only visible cubes or faces.)

1. Students study Santiago's work. Santiago has completed his work incorrectly.

2. Students respond to a question related to Santiago's work:

- How can you tell from the circled cube that Santiago didn't count the number of cubes correctly?

Sample response:

The circled cube should count as one cube and not as 3 different cubes.

3. Students do a similar problem on their own.

Solution:

75 cubic units

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to find the volume of a rectangular prism by counting and using layers.

1. Students study Gabriella's work. Gabriella has completed her work correctly.

2. Students respond to two questions related to Gabriella's work:

- Gabriella counted 12 cubes in the top layer of the prism. How did she get her final answer?

Sample responses:

There are two layers of 12, which is 24.

She multiplied it by 2 because there are two layers.

She added it twice because there are two layers.

- Why is it helpful to use layers to calculate volume?

Sample responses:

It's faster - you need to count fewer cubes.

You can't see all the cubes in the prism.

3. Students do a similar problem on their own.

Solution:

18 cubic inches

SET TWO

1. Study Gabriella's correct work.

✓ Name: Gabriella

Find the volume of the rectangular prism by counting the number of cubes.

= 1 cubic inch

Volume = 24 cubic inches

2. Answer these questions.

- Gabriella counted 12 cubes in the top layer of the prism. How did she get her final answer?
- Why is it helpful to use layers to calculate volume?

3. Then complete this one.

Find the volume of the rectangular prism by counting the number of cubes.

= 1 cubic inch

Name: _____ Date: _____

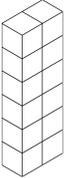
SET ONE

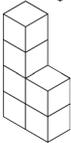
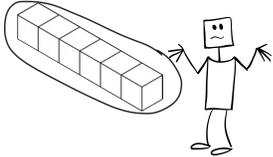
1. Study Claire's incorrect work.

X Name: Claire

Circle **all** shapes with a volume of **6 cubic feet**.

 = 1 cubic foot

a.  b. 

c.  d. 

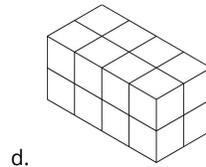
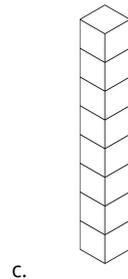
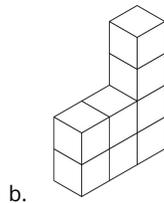
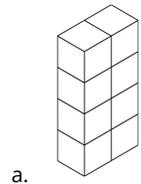
2. Answer these questions.

- Claire correctly circled shape d. Explain how you know that shape d has a volume of 6 cubic feet.
- Claire was supposed to circle all shapes with a volume of 6 cubic feet. Which other shapes should Claire have circled?
- Why can shapes that look different have the same volume?

3. Then complete this one.

Circle **all** shapes with a volume of **8 cubic feet**.

 = 1 cubic foot



Teacher Notes

SET 1 OF 2

TARGET:

*Understanding how to determine the volumes of different shapes.
(Targeted Error: Failing to recognize that different shapes can have the same volume.)*

1. Students study Claire's work. Claire has completed her work incorrectly.

2. Students respond to three questions related to Claire's work:

- Claire correctly circled shape d. Explain how you know that shape d has a volume of 6 cubic feet.

Sample response:

Shape d is made up of 6 cubes, and each cube is equal to one cubic foot.

- Claire was supposed to circle all shapes with a volume of 6 cubic feet. Which other shapes should Claire have circled?

shapes a and c

- Why can shapes that look different have the same volume?

Sample response:

They're made up of the same number of cubes.

3. Students do a similar problem on their own.

Solution:

shapes a, b, and c

Teacher Notes

SET 2 OF 2

TARGET:

Understanding that a rectangular prism can be partitioned in multiple ways to find volume.

1. Students study Landon's work. Landon has completed his work correctly.

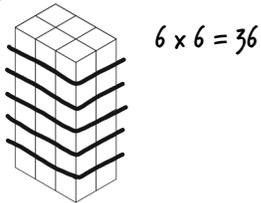
2. Students respond to three questions related to Landon's work:

- Landon partitioned his prism into 3 layers so he did not need to count all the cubes. In the step marked with an arrow, what does 12 represent?

Sample response:

It's the number of cubes in each layer.

- Landon could have partitioned the prism differently. Based on the partitioning below, write another number sentence you could use to find the volume.



- Will the volume always be the same, regardless of how you partition the prism? How do you know?

Sample response:

Yes, the number of total cubes in the prism stays the same.

3. Students do a similar problem on their own.

Solution:

32 cubic feet

SET TWO

1. Study Landon's correct work.

✓ Name: Landon

Find the volume of the prism.

= 1 cubic foot

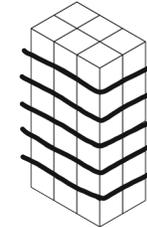
$3 \times 12 = 36$

The volume is 36 cubic feet.

2. Answer these questions.

- Landon partitioned his prism into 3 layers so he did not need to count all the cubes. In the step marked with an arrow, what does 12 represent?

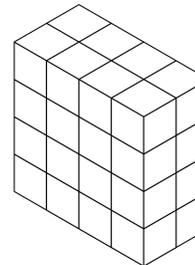
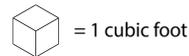
- Landon could have partitioned the prism differently. Based on the partitioning below, write another number sentence you could use to find the volume.



- Will the volume always be the same, regardless of how you partition the prism? How do you know?

3. Then complete this one.

Find the volume of the prism.



Name: _____ Date: _____

SET ONE

1. Study Hailey's correct work.

Name: Hailey

Find the volume of the box by packing it with unit cubes.

= 1 cubic unit

27 cubic units

2. Answer the question.

- How do you know it takes 27 unit cubes to fill the box?

3. Then complete this one.

Find the volume of the box by packing it with unit cubes.

= 1 cubic unit

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to find volume by packing a box with unit cubes.

- Students study Hailey's work. Hailey has completed her work correctly.**

- Students respond to a question related to Hailey's work:**

- How do you know it takes 27 unit cubes to fill the box?

Sample responses:

There are three layers of 9 cubes, and $9 \times 3 = 27$.

$$3 \times 3 \times 3 = 27$$

- Students do a similar problem on their own.**

Solution:

125 cubic units

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to find the volume of a net that is folded into a box.
(Targeted Error: Counting all faces of the net to determine volume.)

1. Students study Hiro's work. Hiro has completed his work incorrectly.

2. Students respond to two questions related to Hiro's work:

- The area of the base is 4 square units. How do you know this?

Sample response:

I know this because the base is in the middle, and there are four boxes in the middle.

- How could Hiro have used the area of the base to conclude that the correct volume is 8 cubic units?

Sample response:

He could have multiplied the area of the base, 4, by the height, 2.

3. Students do a similar problem on their own.

Solution:

16 cubic units

Sample explanation:

There are 8 unit cubes in the base and 2 layers, so $8 \times 2 = 16$ cubic units.

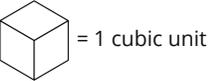
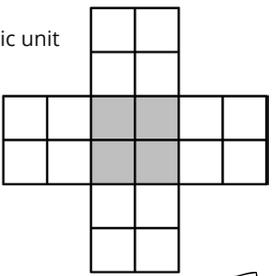
SET TWO

1. Study Hiro's incorrect work.



Name: Hiro

Imagine this net were folded into a box. Find the volume of the box by packing it with unit cubes. Explain your reasoning.

Explanation:

There are 20 faces, so the volume is 20 cubic units.



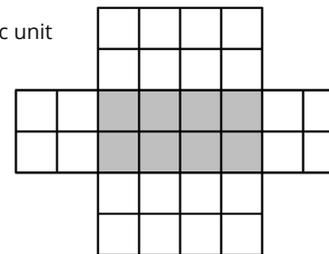
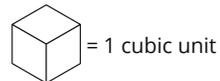
2. Answer these questions.

- The area of the base is 4 square units. How do you know this?

- How could Hiro have used the area of the base to conclude that the correct volume is 8 cubic units?

3. Then complete this one.

Imagine this net were folded into a box. Find the volume of the box by packing it with unit cubes. Explain your reasoning.



Explanation:

SET ONE

1. Study Kaylee's incorrect work.

X

Name: Kaylee

Find the volume of the box by packing it with unit cubes.

$3 \times 5 = 15$ cubic units

2. Answer these questions.

- 1. Kaylee forgot to include one of the dimensions. Why can't you just use two dimensions to find volume?

- 2. By what other number should she have multiplied 3×5 ? Explain how you know.

3. Then complete this one.

Find the volume of the box by packing it with unit cubes.

Geometry/Volume
Packing Unit Cubes (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to find the volume of a box. (Targeted Error: Multiplying only two of the three dimensions to determine volume.)

1. Students study Kaylee's work. Kaylee has completed her work incorrectly.

2. Students respond to two questions related to Kaylee's work:

- 1. Kaylee forgot to include one of the dimensions. Why can't you just use two dimensions to find volume?

Sample responses:

You need 3 dimensions to find volume.

Two dimensions are used to find area.

- 2. By what other number should she have multiplied 3×5 ? Explain how you know.

Sample response:

2, there are 2 layers of 15 unit cubes.

3. Students do a similar problem on their own.

Solution:

12 cubic units

Teacher Notes

SET 2 OF 2

TARGET:

Determining volume by multiplying the area of the base by the height of the box.

1. Students study Connor's work. Connor has completed his work correctly.

2. Students respond to two questions related to Connor's work:

- ⦿ Rather than counting every cube, Connor used multiplication. How did he find the area of the base?

$$4 \times 2$$

- ⦿ Connor multiplied the area of the base by the height. What is the height?

$$2$$

3. Students do a similar problem on their own.

Solution:

6 cubic units

SET TWO

1. Study Connor's correct work.

✓ Name: Connor

Find the volume of the box by packing it with unit cubes.

= 1 cubic unit

2

4 2

$$\begin{array}{r} 4 \\ \times 2 \\ \hline 8 \end{array}$$
$$\begin{array}{r} 8 \\ \times 2 \\ \hline 16 \end{array}$$

16 cubic units

2. Answer these questions.

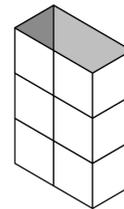
- ⦿ Rather than counting every cube, Connor used multiplication. How did he find the area of the base?

- ⦿ Connor multiplied the area of the base by the height. What is the height?

3. Then complete this one.

Find the volume of the box by packing it with unit cubes.

= 1 cubic unit



Name: _____ Date: _____

SET ONE

1. Study Levi's correct work.

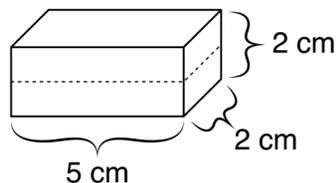
Name: Levi

Find the volume of the prism using a formula.

Volume = $L \times W \times H$
 $= 2\text{cm} \times 2\text{cm} \times 3\text{cm}$
 $= 4\text{cm}^2 \times 3\text{cm}$
 $= 12\text{cm}^3$

3. Then complete this one.

Find the volume of the prism using a formula.



2. Answer these questions.

- 1. How many layers does this prism have?
- 2. Which dimension of the prism gives you that number?
- 3. How many cm^2 make up the base?
- 4. Which dimensions of the prism give you that number?

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to determine the area of the base and the height of a prism.

1. **Students study Levi's work. Levi has completed his work correctly.**
2. **Students respond to four questions related to Levi's work:**
 - 1. How many layers does this prism have?
3
 - 2. Which dimension of the prism gives you that number?
height
 - 3. How many cm^2 make up the base?
4 cm^2
 - 4. Which dimensions of the prism give you that number?
length and width

3. Students do a similar problem on their own.

Solution:

20 cm^3

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to apply the formula $V = L \times W \times H$ to find the volume of a prism. (Targeted Error: Adding the dimensions of the prism instead of multiplying.)

1. Students study Sadie's work. Sadie has completed her work incorrectly.

2. Students respond to two questions related to Sadie's work:

- ☹ Sadie did not use the correct formula for calculating volume. What is the correct formula?

Sample responses:

length \times width \times height

base \times height

- ☹ $4 + 2 + 3$ gives Sadie the total length of those three sides. But this is not the same thing as volume. What does volume measure?

Sample response:

Volume measures the number of unit cubes it takes to fill a prism.

3. Students do a similar problem on their own.

Solution:

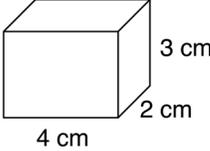
42 cm^3

SET TWO

1. Study Sadie's incorrect work.

X Name: Sadie

Find the volume of the prism using a formula.



Volume = $L + W + H$

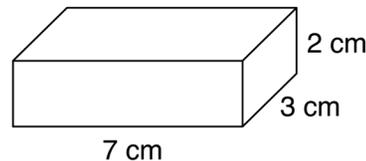
$$\begin{array}{r} 4 \text{ cm} \\ 2 \text{ cm} \\ + 3 \text{ cm} \\ \hline 9 \text{ cm} \end{array}$$


2. Answer these questions.

- ☹ Sadie did not use the correct formula for calculating volume. What is the correct formula?
- ☹ $4 + 2 + 3$ gives Sadie the total length of those three sides. But this is not the same thing as volume. What does volume measure?

3. Then complete this one.

Find the volume of the prism using a formula.



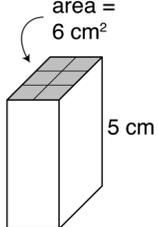
Name: _____ Date: _____

SET ONE

1. Study Aaliyah's correct work.

 Name: Aaliyah

Find the volume of the prism using a formula.



area =
6 cm²

5 cm

Volume = Base × Height
= 6 cm² × 5 cm
= 30 cm³

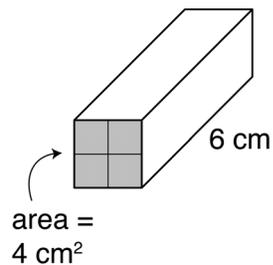


2. Answer the question.

- Another formula for volume is $L \times W \times H$. How was Aaliyah able to determine the correct volume by multiplying the area of the base by the height?

3. Then complete this one.

Find the volume of the prism using a formula.



Geometry/Volume
Volume Formula (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding the relationship between the formula $V = L \times W \times H$ and $V = B \times H$.

1. Students study Aaliyah's work. Aaliyah has completed her work correctly.

2. Students respond to a question related to Aaliyah's work:

- Another formula for volume is $L \times W \times H$. How was Aaliyah able to determine the correct volume by multiplying the area of the base by the height?

Sample response:

$L \times W$ = the area of one of the faces. So, Aaliyah just needs to multiply that area by the height in order to find the volume.

3. Students do a similar problem on their own.

Solution:

24 cm³

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to determine the volume of a box. (Targeted Error: Indicating volume as one dimensional — cm — instead of three dimensional — cm^3 .)

1. Students study Hassan's work. Hassan has completed his work incorrectly.

2. Students respond to a question related to Hassan's work:

- Hassan should have written 6 ft^3 , rather than 6 ft. Why does Hassan need to use feet cubed?

Sample responses:

Volume includes three dimensions.

When you multiply 3 numbers with cm, you have to use cm^3 .

3. Students do a similar problem on their own.

Solution:

The volume of the box is 54 cubic inches (54 in^3).

SET TWO

1. Study Hassan's incorrect work.

X Name: Hassan

Find the volume of the box described using a formula. Write your answer in sentence form.

A boy built a rectangular box to hold his school books. It has a length of 2 feet, a width of 3 feet, and a height of 1 foot. What is the volume of the box?

Volume = $L \times W \times H$
 $= 2\text{ft} \times 3\text{ft} \times 1\text{ft}$
 $= 6\text{ft}$

The volume of the box is 6ft.

2. Answer the question.

- Hassan should have written 6 ft^3 , rather than 6 ft. Why does Hassan need to use feet cubed?

3. Then complete this one.

Find the volume of the box described using a formula. Write your answer in sentence form.

A girl found a box to hold her pencils. The box has a length of 3 inches, a width of 3 inches, and a height of 6 inches. What is the volume of the box?

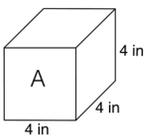
SET ONE

1. Study Hunter's correct work.

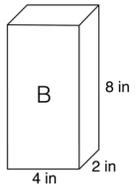


Name: Hunter

Find the volume of each prism.



4 in
4 in
4 in



8 in
4 in
2 in

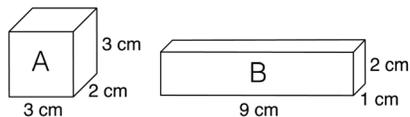
Volume A = $4\text{ in} \times 4\text{ in} \times 4\text{ in}$
 $= 16\text{ in}^2 \times 4\text{ in}$
 $= 64\text{ in}^3$

Volume B = $8\text{ in} \times 4\text{ in} \times 2\text{ in}$
 $= 32\text{ in}^2 \times 2\text{ in}$
 $= 64\text{ in}^3$



3. Then complete this one.

Find the volume of each prism.



2. Answer these questions.

- Hunter found the same volume for both prisms, but the dimensions are not the same. How is this possible?
- List another set of dimensions with a volume of 64 in^3 .
- Draw a prism using your dimensions above.

Teacher Notes

SET 1 OF 2

TARGET:

Understanding that two prisms can have different dimensions but have the same volume.

1. Students study Hunter's work. Hunter has completed his work correctly.

2. Students respond to three questions related to Hunter's work:

- Hunter found the same volume for both prisms, but the dimensions are not the same. How is this possible?

Sample responses:

It still takes 64 cubic inches to fill each prism. The cubes are just arranged differently.

Each set of dimensions multiplied together still equals 64 in^3 .

- List another set of dimensions with a volume of 64 in^3 .

Sample responses:

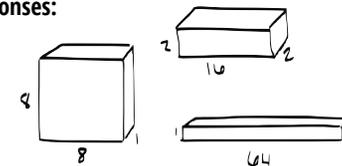
$64 \times 1 \times 1$

$16 \times 2 \times 2$

$8 \times 8 \times 1$

- Draw a prism using your dimensions above.

Sample responses:



3. Students do a similar problem on their own.

Solution:

Volume A = 18 cm^3

Volume B = 18 cm^3

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to find the volume of a shape with unlabeled dimensions. (Targeted Error: Ignoring unlabeled dimensions.)

1. Students study Anka's work. Anka has completed her work incorrectly.

2. Students respond to two questions related to Anka's work:

- How can you tell that Anka did not find the volume of the whole shape?

Sample response:

She only multiplied the dimensions of the prism on the left.

- Anka multiplied $3\text{ft} \times 4\text{ft} \times 7\text{ft}$. What are the dimensions of the whole shape?

$7\text{ft} \times 3\text{ft} \times 8\text{ft}$

3. Students do a similar problem on their own.

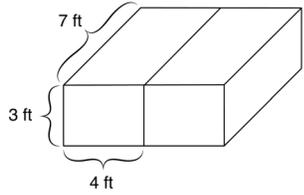
Solution:

160 m^3

SET TWO

1. Study Anka's incorrect work.

X Find the total volume of the shape. The prism on the right is identical to the prism on the left. Name: Anka



Volume = $3\text{ft} \times 4\text{ft} \times 7\text{ft}$
 $= 12\text{ft}^2 \times 7\text{ft}$
 $= 84\text{ft}^3$

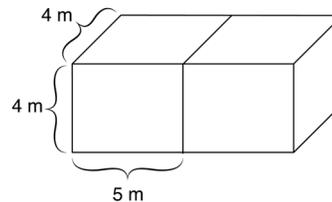


2. Answer these questions.

- How can you tell that Anka did not find the volume of the whole shape?
- Anka multiplied $3\text{ft} \times 4\text{ft} \times 7\text{ft}$. What are the dimensions of the whole shape?

3. Then complete this one.

Find the total volume of the shape. The prism on the right is identical to the prism on the left.



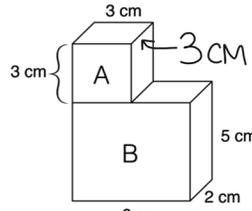
SET ONE

1. Study Jack's incorrect work.

X

Name: Jack

Find the total volume of the shape.



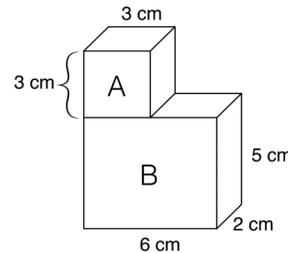
$V_A = 3\text{ cm} \times 3\text{ cm} \times 3\text{ cm} = 27\text{ cm}^3$
 $V_B = 6\text{ cm} \times 2\text{ cm} \times 5\text{ cm} = 60\text{ cm}^3$
 Volume = $27\text{ cm}^3 + 60\text{ cm}^3 = 87\text{ cm}^3$
 87 cm^3



2. Answer these questions.

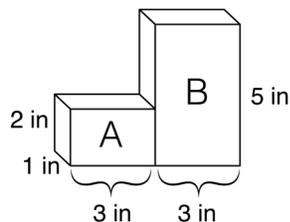
- The diagram shows that the width of prism B is 2 cm. Explain how Jack should have found the width of prism A.

- Circle the five lengths in the shape below with a measurement of 2 cm.



3. Then complete this one.

Find the total volume of the shape.



Geometry/Volume
Prisms with Unlabeled Dimensions

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to find the volume of a shape with unlabeled dimensions. (Targeted Error: Failing to use an equivalent labeled length.)

1. Students study Jack's work. Jack has completed his work incorrectly.

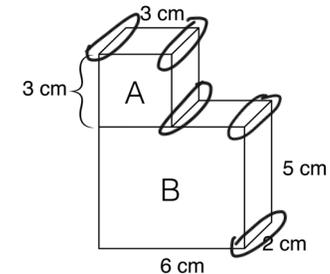
2. Students respond to two questions related to Jack's work:

- The diagram shows that the width of prism B is 2 cm. Explain how Jack should have found the width of prism A.

Sample response:

Prism B has a width of 2 cm, and the two prisms are perfectly lined up. Therefore, the width of prism A must also be 2 cm.

- Circle the five lengths in the shape below with a measurement of 2 cm.



3. Students do a similar problem on their own.

Solution:

21 in^3

Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to find the volume of a shape with unlabeled dimensions by using labeled dimensions.

1. Students study Camila's work. Camila has completed her work correctly.

2. Students respond to two questions related to Camila's work:

- The shape has 7cm as a labeled dimension. Why didn't Camila use 7cm in her calculations?

Sample response:

7cm represents the height of both prisms, and she needed to use the height of each separate prism in her calculations.

- In the step marked with an arrow, how did Camila know to multiply by 2 when there is no "2" on the diagram?

Sample response:

She knew that the total height of the shape was 7, and the height of prism B was 5. So, to find the height of the prism A, she calculated $7 - 5 = 2$.

3. Students do a similar problem on their own.

Solution:

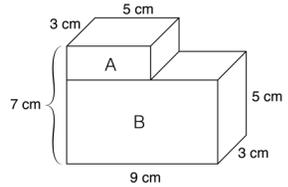
$$26 \text{ ft}^3$$

SET TWO

1. Study Camila's correct work.

Name: Camila

Find the total volume of the shape.



↓

$V_A = 5 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 30 \text{ cm}^3$
 $V_B = 9 \text{ cm} \times 3 \text{ cm} \times 5 \text{ cm} = 135 \text{ cm}^3$
 $\text{Volume} = 30 \text{ cm}^3 + 135 \text{ cm}^3 = 165 \text{ cm}^3$

165 cm³

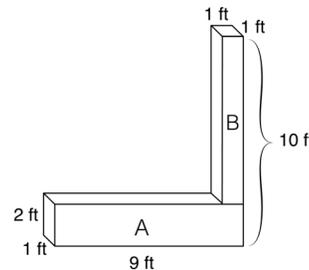
2. Answer these questions.

- The shape has 7cm as a labeled dimension. Why didn't Camila use 7cm in her calculations?

- In the step marked with an arrow, how did Camila know to multiply by 2 when there is no "2" on the diagram?

3. Then complete this one.

Find the total volume of the shape.



Name: _____ Date: _____

SET ONE

1. Study Imani's incorrect work.

X

Name: IMANI

Name the letter located at each coordinate pair.

point	x-coordinate	y-coordinate
A	2	3
C	3	2
E	4	1
D	1	4
B	6	0
F	0	6

2. Answer these questions.

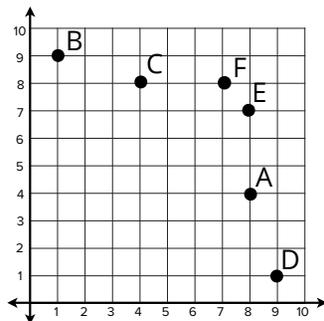
1. Imani wrote the wrong letter for each coordinate pair. What did she confuse?

2. Look at the coordinate plane. Which point has an x-coordinate of 2 and a y-coordinate of 3?

3. Then complete this one.

Name the letter located at each coordinate pair.

point	x-coordinate	y-coordinate
	8	4
	4	8
	1	9
	9	1
	7	8
	8	7



Ordered Pairs

Graphing on the Coordinate Plane (Part 1)

Teacher Notes

SET 1 OF 2

TARGET:

Interpreting points on a coordinate plane. (Targeted Error: Reversing the x- and y-coordinates.)

1. Students study Imani's work. Imani has completed her work incorrectly.

2. Students respond to two questions related to Imani's work:

1. Imani wrote the wrong letter for each coordinate pair. What did she confuse?

Sample response:

She switched the x- and y-coordinates.

2. Look at the coordinate plane. Which point has an x-coordinate of 2 and a y-coordinate of 3?

point C

3. Students do a similar problem on their own.

Solution:

point	x-coordinate	y-coordinate
A	8	4
C	4	8
B	1	9
D	9	1
F	7	8
E	8	7

Teacher Notes

SET 2 OF 2

TARGET:

Understanding why the ordering of the numbers in an ordered pair changes the position of the point on the coordinate plane.

1. Students study Chung's work. Chung has completed his work correctly.

2. Students respond to a question related to Chung's work:

- Why are (5, 3) and (3, 5) different points on the coordinate plane even though they are made up of the same numbers?

Sample response:

The first number is the x-coordinate, and the second number is the y-coordinate.

3. Students do a similar problem on their own.

Solution:

point	x-coordinate	y-coordinate
A	2	3
B	7	3
C	2	1
D	7	1

SET TWO

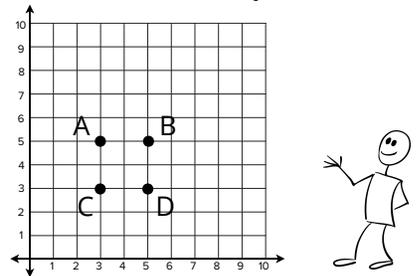
1. Study Chung's correct work.



Write the coordinates for each letter.

point	x-coordinate	y-coordinate
A	3	5
B	5	5
C	3	3
D	5	3

Name: Chung



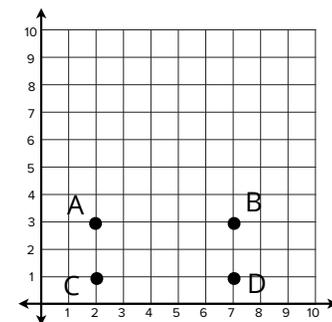
2. Answer the question.

- Why are (5, 3) and (3, 5) different points on the coordinate plane even though they are made up of the same numbers?

3. Then complete this one.

Write the coordinates for each letter.

point	x-coordinate	y-coordinate
A		
B		
C		
D		



Name: _____ Date: _____

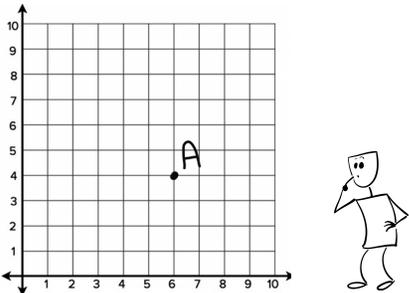
SET ONE

1. Study Evan's incorrect work.

X Use the coordinate plane to complete the tasks.

- Plot and label the following point.
A (4, 6)
- What steps did you take to plot the point?
I went up 4 and to the right 6

Name: Evan



2. Answer these questions.

🗣️ Is 4 the x-coordinate or the y-coordinate?

🗣️ Starting at the origin, which direction should Evan have moved when plotting the x-coordinate?

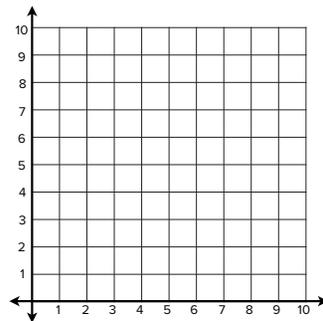
3. Then complete this one.

Use the coordinate plane to complete the tasks.

Plot and label the following point.

B (6, 3)

What steps did you take to plot the point?



Ordered Pairs

Graphing on the Coordinate Plane (Part 2)

Teacher Notes

SET 1 OF 2

TARGET:

Plotting points on a coordinate plane. (Targeted Error: Reversing the x- and y-coordinates.)

1. Students study Evan's work. Evan has completed his work incorrectly.

2. Students respond to two questions related to Evan's work:

🗣️ Is 4 the x-coordinate or the y-coordinate?

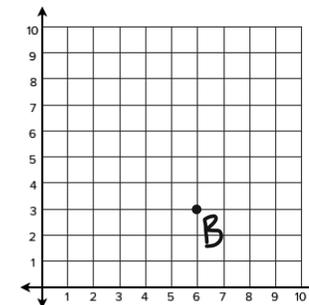
x-coordinate

🗣️ Starting at the origin, which direction should Evan have moved when plotting the x-coordinate?

to the right

3. Students do a similar problem on their own.

Solution:



I went to the right 6 and up 3.

Ordered Pairs
Graphing on the Coordinate Plane (Part 2), continued

Teacher Notes

SET 2 OF 2

TARGET:

Understanding that coordinate pair (a, b) is not the same as coordinate pair (b, a) .

1. Students study Madelyn's work. Madelyn has completed her work correctly.

2. Students respond to a question related to Madelyn's work:

- When writing the ordered pair for point A, could Madelyn have also written $(5, 3)$? Why or why not?

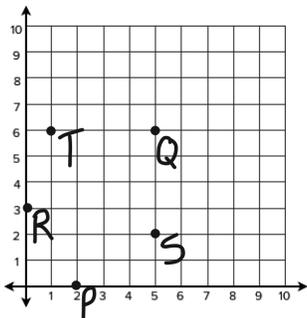
Sample response:

No, the x-coordinate is always written first.

3. Students do a similar problem on their own.

Solution:

point	x-coordinate	y-coordinate	ordered pair
P	2	0	P (2, 0)
Q	5	6	Q (5, 6)
R	0	3	R (0, 3)
S	5	2	S (5, 2)
T	1	6	T (1, 6)



Ordered Pairs
Graphing on the Coordinate Plane (Part 2), continued

SET TWO

1. Study Madelyn's correct work.

Write the ordered pairs in the chart. Then plot and label the points.

point	x-coordinate	y-coordinate	ordered pair
A	3	5	(3, 5)
B	2	4	(2, 4)
C	1	2	(1, 2)
D	4	4	(4, 4)
E	5	0	(5, 0)

Name: Madelyn

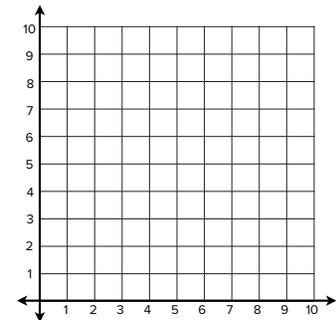
2. Answer the question.

- When writing the ordered pair for point A, could Madelyn have also written $(5, 3)$? Why or why not?

3. Then complete this one.

Write the ordered pairs in the chart. Then plot and label the points.

point	x-coordinate	y-coordinate	ordered pair
P	2	0	
Q	5	6	
R	0	3	
S	5	2	
T	1	6	



Name: _____ Date: _____

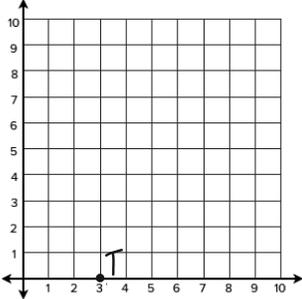
SET ONE

1. Study Caroline's incorrect work.

X Plot and label the following point.

T (0, 3)

Name: Caroline



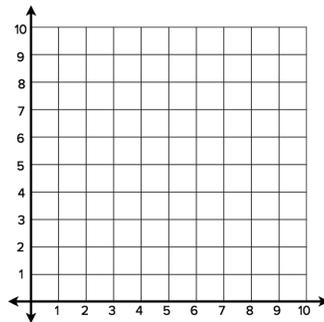
2. Answer these questions.

- 🗣️ In order to plot (0, 3), how many units **to the right** should Caroline have moved from the origin? Explain how you know.
- 🗣️ How many units **up** should Caroline have moved? Explain how you know.

3. Then complete this one.

Plot and label the following point.

G (4, 0)



Ordered Pairs

Graphing on the Coordinate Plane (Part 3)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to plot a point when 0 is a coordinate. (Targeted Error: Plotting the point with 0 as a coordinate on the wrong axis.)

1. Students study Caroline's work. Caroline has completed her work incorrectly.

2. Students respond to two questions related to Caroline's work:

- 🗣️ In order to plot (0, 3), how many units **to the right** should Caroline have moved from the origin? Explain how you know.

Sample response:

0 units to the right, that is the x-coordinate.

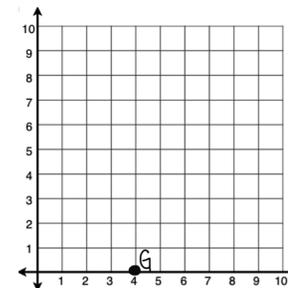
- 🗣️ How many units **up** should Caroline have moved? Explain how you know.

Sample response:

3 units up, that is the y-coordinate.

3. Students do a similar problem on their own.

Solution:



Teacher Notes

SET 2 OF 2

TARGET:

Understanding that coordinate pair (a, b) is not the same as coordinate pair (b, a) .

1. Students study Adrian's work. Adrian has completed her work correctly.

2. Students respond to a question related to Adrian's work:

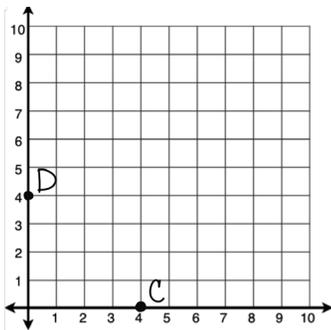
- If Adrian had put point B on the y -axis and point A on the x -axis, would he still have been correct? How do you know?

Sample response:

No, because in point A, 9 is the y -coordinate, and in point B, 9 is the x -coordinate. They cannot be switched.

3. Students do a similar problem on their own.

Solution:



SET TWO

1. Study Adrian's correct work.

Plot and label the following points.

A (0, 9)

B (9, 0)

Name: Adrian

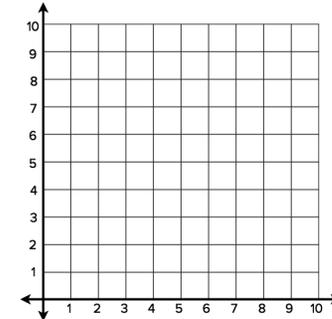
2. Answer the question.

- If Adrian had put point B on the y -axis and point A on the x -axis, would he still have been correct? How do you know?

3. Then complete this one.

Plot and label the following points.

- C (4, 0)
- D (0, 4)



Name: _____ Date: _____

SET ONE

1. Study Nick's incorrect work.

X

Plot and label the points below.

point	ordered pair
A	$(4, 4\frac{1}{2})$
B	$(0, \frac{1}{2})$
C	$(2\frac{1}{2}, 1\frac{1}{2})$
D	$(1\frac{1}{2}, 3)$

Name: Nick

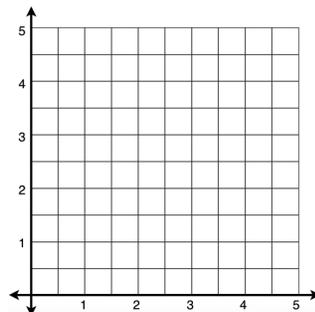
2. Answer these questions.

- 🗣️ What is the scale used on the coordinate plane? How can you tell?
- 🗣️ Knowing the scale used on the coordinate plane, how can you tell that Nick did not plot the points correctly?

3. Then complete this one.

Plot and label the points below.

point	ordered pair
A	$(3, 3\frac{1}{2})$
B	$(\frac{1}{2}, 0)$
C	$(1\frac{1}{2}, 4\frac{1}{2})$
D	$(3\frac{1}{2}, 1)$



Ordered Pairs

Graphing on the Coordinate Plane (Part 4)

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to plot points on a coordinate plane when intervals represent a value less than 1. (Targeted Error: Misinterpreting the scale of the coordinate plane.)

- 1. Students study Nick's work. Nick has completed his work incorrectly.**
- 2. Students respond to two questions related to Nick's work:**

- 🗣️ What is the scale used on the coordinate plane? How can you tell?

Sample response:

$\frac{1}{2}$, because there is a line between each number on the axes.

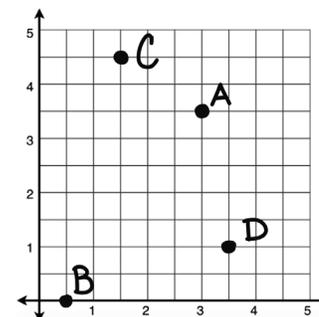
- 🗣️ Knowing the scale used on the coordinate plane, how can you tell that Nick did not plot the points correctly?

Sample response:

Nick plotted points that were halfway between the lines, not on the lines. The lines are every $\frac{1}{2}$, so he should have plotted on the lines.

3. Students do a similar problem on their own.

Solution:



Teacher Notes

SET 2 OF 2

TARGET:

Determining the scale of a coordinate plane when intervals represent a value less than 1.

- Students study Annabelle's work. Annabelle has completed her work correctly.
- Students respond to two questions related to Annabelle's work:

- Why did Annabelle use fractions for some of the coordinates?

Sample responses:

The points didn't lay on a whole number.

Each whole number is 2 units away from the next whole number.

- How did Annabelle know to use $\frac{1}{2}$ in her coordinates instead of another fraction?

Sample responses:

There is a line between each whole number, so each of the lines represents $\frac{1}{2}$.

That is the scale used on the coordinate plane.

- Students do a similar problem on their own.

Solution:

point	ordered pair
A	(1, 1)
B	(1, 2.5)
C	(3, 3.5)
D	(2.5, 0.5)

SET TWO

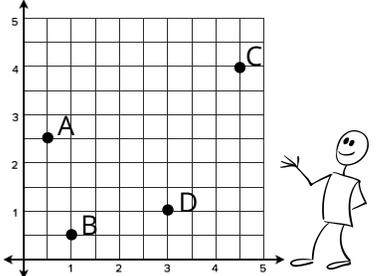
- Study Annabelle's correct work.



Write the ordered pair for each point in the chart below.

Name: Annabelle

point	ordered pair
A	$(\frac{1}{2}, 2\frac{1}{2})$
B	$(1, \frac{1}{2})$
C	$(4\frac{1}{2}, 4)$
D	$(3, 1)$



- Answer these questions.

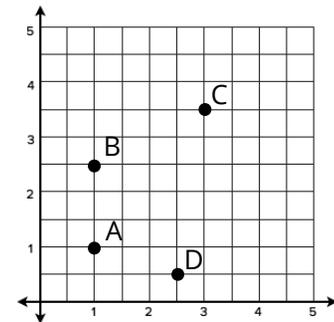
- Why did Annabelle use fractions for some of the coordinates?

- How did Annabelle know to use $\frac{1}{2}$ in her coordinates instead of another fraction?

- Then complete this one.

Write the ordered pair for each point in the chart below.

point	ordered pair
A	
B	
C	
D	



Name: _____ Date: _____

SET ONE

1. Study Melanie's incorrect work.

X

Name: Melanie

Use the story to plot a line graph.

The number of 5th grade students in a school has changed over time. The school opened in 2000 with 200 5th graders. The 5th grade class grew by 10 students each year before reaching its largest class of 290 students in 2009. In 2010, the class decreased by 10 students.

Year	Number of 5th graders
2000	200
2001	210
2002	220
2003	230
2004	240
2005	250
2006	260
2007	270
2008	280
2009	290
2010	280

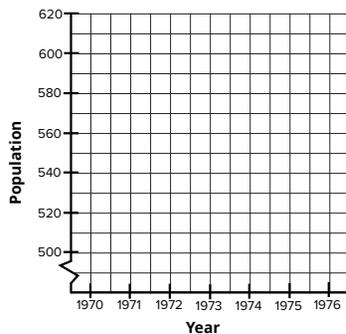
2. Answer these questions.

- 1. Melanie correctly graphed the points between 2000 and 2009. However, she forgot to plot a point at 2010. What part of the word problem indicates that Melanie needed to plot this point?
- 2. What is the ordered pair for the year 2010?

3. Then complete this one.

Use the story to plot a line graph.

The population of a small town has changed over the years. In 1970, 600 people lived in the town. The population decreased by 20 people each year before reaching its low of 500 people in 1975. However, the following year the population increased by 20 people.



Ordered Pairs
Sketching Line Graphs

Teacher Notes

SET 1 OF 2

TARGET:

Sketching a line graph based on a word problem. (Targeted Error: Not attending to all of the necessary information in a word problem.)

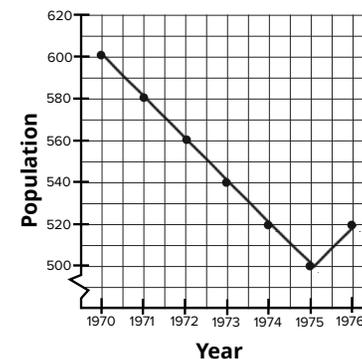
1. Students study Melanie's work. Melanie has completed her work incorrectly.

2. Students respond to two questions related to Melanie's work:

- 1. Melanie correctly graphed the points between 2000 and 2009. However, she forgot to plot a point at 2010. What part of the word problem indicates that Melanie needed to plot this point?
"In 2010, the class decreased by 10 students."
- 2. What is the ordered pair for the year 2010?
(2010, 280)

3. Students do a similar problem on their own.

Solution:



Teacher Notes

SET 2 OF 2

TARGET:

Interpreting a word problem and determining an appropriate scale to plot the data.

1. Students study Felipe's work. Felipe has completed his work correctly.
2. Students respond to a question related to Felipe's work:

Why was it helpful for Felipe to mark every 50 students when writing the numbers on the y-axis?

Sample responses:

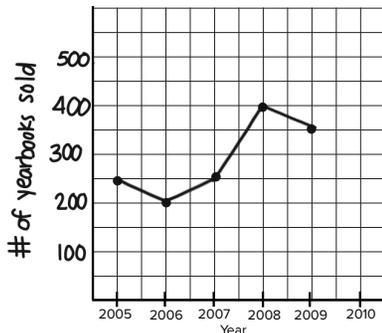
Felipe knew that the numbers had to be evenly spread out on the coordinate plane, and he needed to cover numbers 50 to 200.

Since he used 2 lines for each multiple of 50, it was easy to plot both 175 and 75.

Felipe had to include numbers from 1 to 200 on the y-axis. There were only 13 lines to use, and he needed to spread the numbers out evenly.

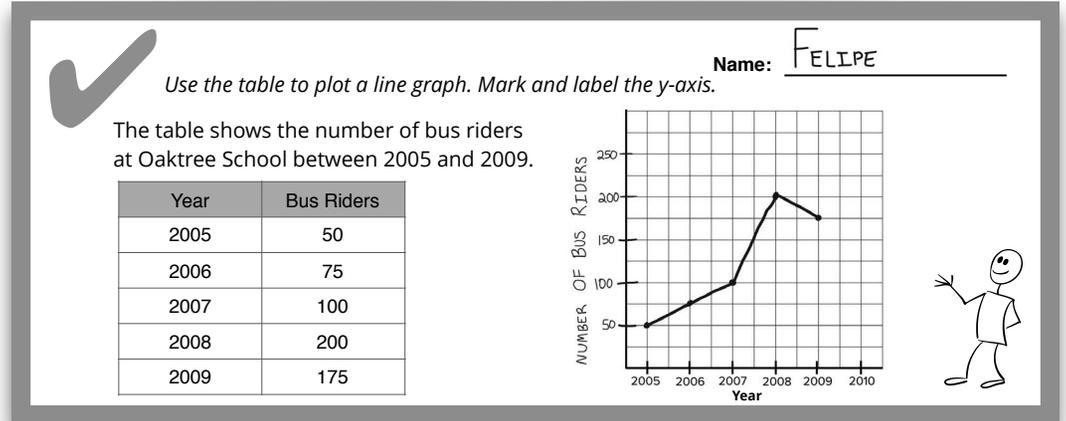
3. Students do a similar problem on their own.

Solution:



SET TWO

1. Study Felipe's correct work.



2. Answer the question.

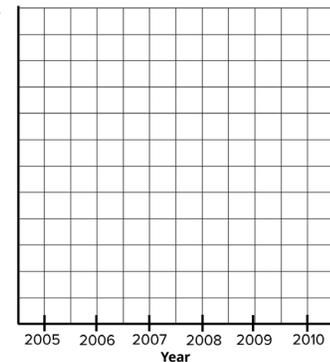
Why was it helpful for Felipe to mark every 50 students when writing the numbers on the y-axis?

3. Then complete this one.

Use the table to plot a line graph. Mark and label the y-axis.

The table shows the number of yearbooks sold at Oaktree School between 2005 and 2009.

Year	Yearbooks
2005	250
2006	200
2007	250
2008	400
2009	350



Name: _____ Date: _____

SET ONE

1. Study Adam's incorrect work.

X Use the graph to answer the questions.

A girl scout is selling candy bars this week.

Name: Adam

- On which day did she sell the least number of candy bars?
Wednesday
- On which day did she sell the greatest number of candy bars?
Thursday
- Between which two consecutive days was the biggest decrease in sales?
Wednesday and Thursday

Day of the Week	Number of Candy Bars Sold
Su	16
M	12
Tu	8
W	4
Th	24
F	6
Sa	8

2. Answer the question.

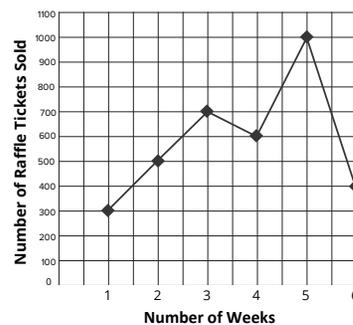
- Adam thought that the biggest decrease was between Wednesday and Thursday because those points are the farthest apart from each other. When was the actual biggest decrease? Explain how you know.

3. Then complete this one.

Use the graph to answer the questions.

The music program is selling raffle tickets for 6 weeks.

- During which week did the music program sell the least number of raffle tickets?
- During which week did the music program sell the greatest number of raffle tickets?
- Between which two consecutive weeks was the biggest decrease in sales?



Ordered Pairs
Interpreting Line Graphs

Teacher Notes

SET 1 OF 2

TARGET:

Interpreting a line graph. (Targeted Error: Associating the biggest decrease with the biggest change without considering change in value.)

1. Students study Adam's work. Adam has completed his work incorrectly.

2. Students respond to a question related to Adam's work:

- Adam thought that the biggest decrease was between Wednesday and Thursday because those points are the farthest apart from each other. When was the actual biggest decrease? Explain how you know.

Sample response:

The biggest decrease was really between Thursday and Friday. That's when the graph went down the most over one day.

3. Students do a similar problem on their own.

Solution:

- week 1
- week 5
- weeks 5 and 6

Teacher Notes

SET 2 OF 2

TARGET:

Interpreting a line graph to solve problems. (Targeted Error: Using the wrong operation to solve a problem.)

1. Students study Naomi's work. Naomi has completed her work correctly.

2. Students respond to a question related to Naomi's work:

- Why did Naomi decide to subtract, rather than add, in order to answer the second question?

Sample response:

Naomi subtracted because she was trying to find the difference between the two months.

3. Students do a similar problem on their own.

Solution:

- \$60
- \$20

SET TWO

1. Study Naomi's correct work.

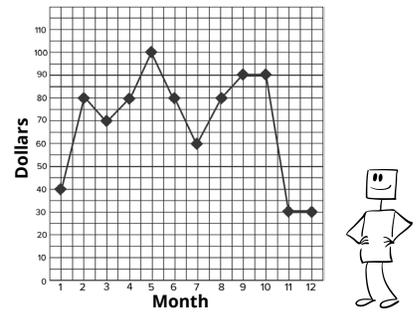


Use the graph to answer the questions.

The graph shows the total amount of money in a student's bank account at the end of each month.

- How much money did the student have in her account at the end of the 1st month?
\$40
- How much more money did the student have at the end of the 6th month than at the end of the 7th month?
 $\$80 - \$60 = \$20$

Name: Naomi



2. Answer the question.

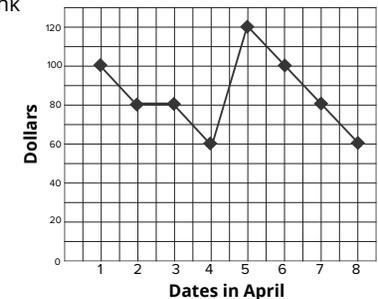
- Why did Naomi decide to subtract, rather than add, in order to answer the second question?

3. Then complete this one.

Use the graph to answer the questions.

The graph shows the total amount of money in a man's bank account between April 1st and April 8th.

- How much money did the man have on April 4th?
- How much more money did the man have on the 6th than on the 7th?



Name: _____ Date: _____

SET ONE

1. Study Eleanor's correct work.

Name: Eleanor

Write a rule that describes the relationship between the x - and y -coordinates.

x	y
4	2
5	3
7	5
8	6

Rule: x is 2 more than y

2. Answer these questions.

- Ⓐ Could Eleanor have used the rule “ x is 2 times y ” instead? Explain your reasoning.
- Ⓑ Why is it important to look at all of the rows in the data table?

3. Then complete this one.

Write a rule that describes the relationship between the x - and y -coordinates.

x	y
0	3
2	5
4	7
6	9

Rule: _____

Ordered Pairs
Generating Number Patterns

Teacher Notes

SET 1 OF 2

TARGET:

Determining a rule to describe the relationship between x - and y -coordinates. (Targeted Error: Looking at only one ordered pair to determine a rule.)

1. Students study Eleanor's work. Eleanor has completed her work correctly.

2. Students respond to two questions related to Eleanor's work:

- Ⓐ Could Eleanor have used the rule “ x is 2 times y ” instead? Explain your reasoning.

Sample response:

No, x is not always 2 times y . It works for the first line but not the others.

- Ⓑ Why is it important to look at all of the rows in the data table?

Sample responses:

Multiple rules can apply to one set of ordered pairs/coordinates.

The rule needs to work for all ordered pairs.

3. Students do a similar problem on their own.

Sample solutions:

x is 3 less than y

y is 3 more than x

Teacher Notes

SET 2 OF 2

TARGET:

Determining a rule to describe the relationship between x - and y -coordinates. (Targeted Error: Switching the x - and y -coordinates when creating a rule.)

1. Students study Chase's work. Chase has completed his work incorrectly.

2. Students respond to two questions related to Chase's work:

- How can you tell that Chase's rule is incorrect?

Sample response:

When you insert the coordinate pairs, the rule isn't true.

- How could you change the rule to make it correct?

Sample responses:

You could switch the x - and y -coordinates (x is 2 times y).

You could switch the 2 to $\frac{1}{2}$ (y is $\frac{1}{2}$ times x).

3. Students do a similar problem on their own.

Sample solutions:

y is 4 times x

x is $\frac{1}{4}$ times y

SET TWO

1. Study Chase's incorrect work.

X

Name: Chase

Write a rule that describes the relationship between the x - and y -coordinates.

x	y
4	2
8	4
12	6
16	8

Rule: y is 2 times x



2. Answer these questions.

- How can you tell that Chase's rule is incorrect?

- How could you change the rule to make it correct?

3. Then complete this one.

Write a rule that describes the relationship between the x - and y -coordinates.

x	y
1	4
2	8
3	12
4	16

Rule:

Name: _____ Date: _____

Teacher: _____

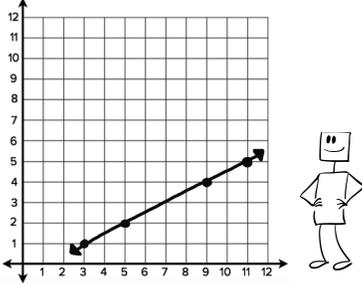
SET ONE

1. Study Ian's correct work.

 Complete the table using the rule. Then graph the line.
x is 1 more than 2 times y

x	y
3	$1 \times 2 + 1$
5	$2 \times 2 + 1$
9	$4 \times 2 + 1$
11	$5 \times 2 + 1$

Name: Ian



2. Answer these questions.

1. Why did Ian write the same expression after each y-coordinate?

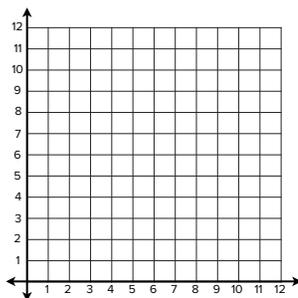
2. What would the x-coordinate be for $y = 10$? How do you know?

3. Then complete this one.

Complete the table using the rule. Then graph the line.

x is 3 less than 2 times y

x	y
	2
	3
	5
	6



Ordered Pairs

Graphing Number Patterns

Teacher Notes

SET 1 OF 2

TARGET:

Understanding how to apply a rule to determine a second coordinate and graphing the coordinate pairs.

1. Students study Ian's work. Ian has completed his work correctly.

2. Students respond to two questions related to Ian's work:

1. Why did Ian write the same expression after each y-coordinate?

Sample response:

The same rule has to be applied to find each ordered pair.

2. What would the x-coordinate be for $y = 10$? How do you know?

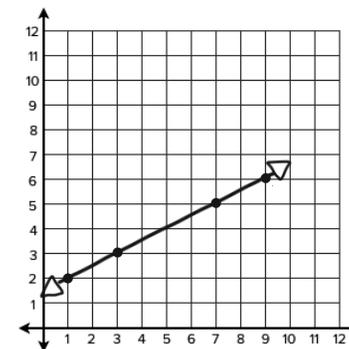
Sample response:

21, because you just insert 10 as the y-coordinate: $10 \times 2 + 1$.

3. Students do a similar problem on their own.

Solution:

x	y
1	2
3	3
7	5
9	6



Teacher Notes

SET 2 OF 2

TARGET:

Understanding how to apply a rule to determine a second coordinate and graph the coordinate pairs. (Targeted Error: Ignoring the order of operations when applying the rule.)

1. Students study Taylor's work. Taylor has completed her work incorrectly.

2. Students respond to a question related to Taylor's work:

- Taylor subtracted 4 from x and then multiplied by 2. Explain why her method was incorrect.

Sample responses:

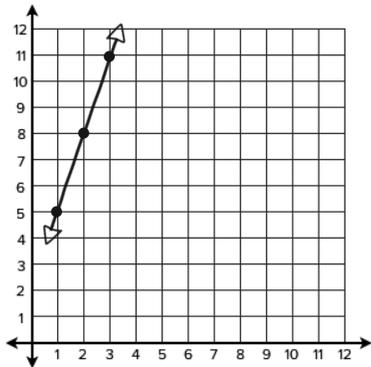
She should have multiplied by 2 and then subtracted 4.

She didn't follow the order of operations.

3. Students do a similar problem on their own.

Solution:

x	y
1	5
2	8
3	11



SET TWO

1. Study Taylor's incorrect work.

X

Complete the table using the rule. Then graph the line.

y is 4 less than 2 times x

x	y
4	0
5	2
8	8

$4 - 4 \times 2 = 0$
 $5 - 4 \times 2 = 2$
 $8 - 4 \times 2 = 8$

Name: Taylor

2. Answer the question.

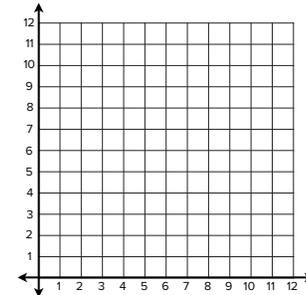
- Taylor subtracted 4 from x and then multiplied by 2. Explain why her method was incorrect.

3. Then complete this one.

Complete the table using the rule. Then graph the line.

y is 2 more than 3 times x

x	y
1	
2	
3	



Appendix: Detailed Assignment Descriptions with Targets

Multi-Digit Numbers

Page	Assignment	Set	Target	Targeted Error	CCSS
1	Multiplying Multi-Digit Numbers (Part 1)	1	Understanding partial products in the standard algorithm.		5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm
		2	<i>see above</i>	Using single-digit multiplication rules when multiplying multi-digit numbers.	
3	Multiplying Multi-Digit Numbers (Part 2)	1	Understanding how to carry the value of 10 when using the standard algorithm to multiply.		<i>see above</i>
		2	Understanding how to use the standard algorithm to multiply.	Ignoring place value of digits in the tens and hundreds column for one factor when multiplying multi-digit numbers.	
5	Dividing Multi-Digit Numbers (Part 1)	1	Understanding how to find quotients using knowledge of multiplication.		5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and /or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and /or area models.
		2	Understanding how to use patterns of 10 when dividing.	Shifting the wrong number of place values.	
7	Dividing Multi-Digit Numbers (Part 2)	1	Understanding how to use an area model to illustrate division.		<i>see above</i>
		2	Understanding how to divide using long division.	Forgetting to note "0" in a multi-digit quotient.	
9	Multi-Digit Numbers Word Problems	1	Understanding how to identify the operation needed to solve a word problem.	Misinterpreting the word problem and using the wrong operation.	5.NBT.B.5 AND 5.NBT.B.6
		2	<i>see above</i>		

Place Value

Page	Assignment	Set	Target	Targeted Error	CCSS
11	Multiplying by Powers of 10	1	Understanding how the digits shift when you multiply by a power of 10.		5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
		2	<i>see above</i>	Misinterpreting the relationship between the power of 10 and the number of digits shifted.	

Place Value, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
13	Dividing by Powers of 10	1	Understanding how the digits shift when you divide by a power of 10.	Shifting the digits in the wrong direction – multiplying instead of dividing.	see above
		2	see above		
15	Understanding Exponents	1	Understanding that 10^x means 10 multiplied by itself x number of times.	Believing that 10^x means 10 multiplied by x.	5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
		2	Understanding the relationship between the number of zeros and the values of the exponents when working with the power of 10.		
17	Multiplying and Dividing by Powers of 10 (Exponents)	1	Understanding the relationship between the number of digits to shift and the exponent when working with the power of 10.		see above
		2	see above	Adding zeros after a decimal number when multiplying by a power of 10, without shifting the digits.	

Decimals

Page	Assignment	Set	Target	Targeted Error	CCSS
19	Writing Decimals	1	Understanding how to convert a number in standard form to word form.	Ignoring the decimal point when stating the word form of a number.	5.NBT.A.3 Read, write, and compare decimals to thousandths: Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.
		2	Understanding the link between place value and decimal form.		
21	Comparing Decimals	1	Understanding how to compare decimals in standard form.	Comparing the 'length' or number of digits in the decimal rather than using knowledge of place value to compare values.	see above
		2	see above	Identifying a decimal with zeros placed to the right of the decimal number as having a different value than decimals without the zeros.	

Decimals, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
23	Rounding Decimals (Part 1)	1	Understanding how to use the number line to aid in rounding.		5.NBT.A.4 Use place value understanding to round decimals to any place.
		2	<i>see above</i>	Changing the value of the digit in the designated place when rounding down.	
25	Rounding Decimals (Part 2)	1	Understanding how to round decimal numbers.	When rounding decimal numbers, including the values to the right of the digit in the designated place.	<i>see above</i>
		2	<i>see above</i>	Rounding to the tens place, instead of to the nearest tenth.	
27	Adding Decimals (Part 1)	1	Understanding how to add whole numbers and decimal numbers.	Adding without considering place value.	5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
		2	Understanding how to apply the commutative property to simplify the work of addition.		
29	Adding Decimals (Part 2)	1	Understanding how to regroup when adding decimals.	Adding without regrouping.	<i>see above</i>
		2	Understanding the properties of place value (e.g., 10 hundredths = 1 tenths).		
31	Subtracting Decimals (Part 1)	1	Understanding how to use concrete models to subtract decimals.		<i>see above</i>
		2	Understanding how to subtract decimals.	Subtracting decimals without considering place value.	
33	Subtracting Decimals (Part 2)	1	Understanding how to regroup when subtracting decimals.		<i>see above</i>
		2	<i>see above</i>	Subtracting the smaller value from the larger value regardless of order.	

Decimals, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
35	Writing Numerical Expressions	1	Determining which operation to use based on the written expression.	Using the wrong operation.	5.OA.A.1- Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.; 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.
		2	Understanding how to convert a written expression to a numerical expression.	Misplacing parentheses.	
37	Writing Numerical Expressions from Word Problems	1	Understanding the commutative property of multiplication.		<i>see above</i>
		2	Understanding how to write a numerical expression based on a word problem.	Applying the concept of "write the numbers you hear" when writing numbers in standard form.	
39	Multiplying Decimals	1	Understanding the role of place value when multiplying decimals.	Multiplying both sides of the decimal point individually by the other factor.	5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
		2	<i>see above</i>	Not knowing where to put the decimal point in the product.	
41	Modeling Multiplying Decimals	1	Understanding how to multiply whole numbers and decimals using an area model.	Decomposing a decimal incorrectly.	<i>see above</i>
		2	Understanding the relationship between multiplication and repetitive addition.		
43	Dividing Decimals Using Place Value	1	Understanding how to use place value to divide decimal numbers		<i>see above</i>
		2	<i>see above</i>	Misunderstanding scale when dividing by a larger number.	
45	Dividing Decimals	1	Misbelieving that a remainder and a decimal quotient are the same.	Carrying over the remaining value without considering the divisor.	<i>see above</i>
		2	Understanding how to represent a division word problem using a tape diagram.		

Decimals, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
47	Converting Units	1	Understanding scale when converting from one unit to another.	Using the wrong scale factor.	5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05m), and use these conversion in solving multi-step, real world problems.
		2	Understanding when to multiply or divide using the conversion factor.		
49	Converting Units Word Problems	1	Understanding how to interpret a word problem involving unit conversion.	Forgetting to convert the units when a word problem asks for a different unit than what is provided.	see above
		2	Understanding the order of operations in the context of solving word problems.		

Basic Operations with Fractions

Page	Assignment	Set	Target	Targeted Error	CCSS
51	Fractions as Division	1	Understanding how division number sentences relate to fractions.	Dividing the larger number by the smaller number, regardless of order.	5.NF.B.3- Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
		2	Understanding that multiple diagrams can be used to model a division problem.		
53	Fraction as Division Word Problems	1	Understanding how to interpret a word problem for division.	Using the wrong operation.	see above
		2	see above	Misinterpreting a division word problem and switching the dividend and divisor.	

Basic Operations with Fractions, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
55	Adding Fractions with Unlike Denominators	1	Understanding why equivalent fractions with a common denominator must be found before adding fractions.		5.NF.A.1- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)
		2	Understanding that in order to add two fractions they must have a common denominator.	Adding the denominators together.	
57	Addition Word Problems	1	Understanding why two different fractions can have the same value.		
		2	Understanding how to find common denominators when adding fractions.	Adding, instead of multiplying, the same number to the numerator and denominator in order to find equivalent fractions.	
59	Subtracting Fractions with Unlike Denominators	1	Understanding how to use tape diagrams to find equivalent fractions and solve a subtraction fraction problem.		5.NF.A.2- Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.
		2	see <i>above</i>	Subtracting the numerators and using the denominator in the first fraction in the answer when subtracting fractions.	
61	Subtraction Word Problems	1	Understanding how to find common denominators in order to subtract fractions.	Multiplying the denominators to find a common denominator without multiplying the numerators to find equivalent fractions.	see <i>above</i>
		2	Interpreting a word problem that includes more than two numbers.		
63	Using Benchmark Fractions to Estimate Answers (Part 1)	1	Understanding how to use 1 as a benchmark to estimate the answer.		see <i>above</i>
		2	Understanding how to correctly use $\frac{1}{2}$ as a benchmark to estimate the answer.		

Basic Operations with Fractions

Page	Assignment	Set	Target	Targeted Error	CCSS
65	Using Benchmark Fractions to Estimate Answers (Part 2)	1	Understanding that there are multiple ways to use mental math to compare fractions in different forms.		<i>see above</i>
		2	Using mental math to compare fractions in different forms.	Believing that all fractions are smaller than mixed numbers, regardless of value.	
67	Fraction Line Plots (Part 1)	1	Understanding how to measure more or less precisely.		5.MD.B.2- Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
		2	Understanding how to measure and indicate rounded values on a line plot.	Rounding to the wrong unit.	
69	Fraction Line Plots (Part 2)	1	Understanding how to determine frequency from a line plot.		<i>see above</i>
		2	Understanding how to measure and indicate rounded values on a line plot.	Measuring the rod using only the "end point" of the rod, without looking to see where the rod began.	
71	Addition and Subtraction Expressions with Fractions	1	Understanding how to interpret the part/whole relationship when the expression is written in words.		5.OA.A.1- Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.; 5.OA.A.2- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.
		2	Understanding how to interpret a written expression.	Not including parentheses.	
73	Multiplication and Division Expressions with Fractions	1	Understanding that there are multiple ways to write a numerical expression based on the same written expression.		<i>see above</i>
		2	Understanding how to interpret a written expression.	Solving without considering order of operations.	

Multiplying and Dividing Fractions

Page	Assignment	Set	Target	Targeted Error	CCSS
75	Multiplying a Fraction by a Whole Number	1	Interpreting the product $(a/b) \times q$ as "a" parts of a partition of "q" into "b" equal parts.	Representing the division problem but not knowing how to determine the value of each part of the partition.	5.NF.B.4.A Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
		2	see above		
77	Multiplying a Fraction by a Fraction	1	Understanding how to multiply fractions.	Applying the procedure of adding fractions—finding a common denominator and using that denominator in the answer—when multiplying fractions.	see above
		2	Understanding how to create a word problem to represent a number sentence.		
79	Area Problems (Part 1)	1	Understanding how an area model breaks down $a \cdot d$ e/f to $(a \cdot d) + (a \cdot e/f)$.		5.NF.B.4.B Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
		2	Understanding how to multiply whole numbers and fractions.	Placing the same denominator of the fraction under the whole number when adding whole numbers and fractions.	
81	Area Problems (Part 2)	1	Understanding how to multiply mixed numbers to find the area of a rectangle.	Writing cm instead of cm^2 when finding an area.	see above
		2	Understanding how an area model breaks down a $b/c \cdot d$ e/f to $(a \cdot d) + (a \cdot e/f) + (b/c \cdot d) + (b/c \cdot e/f)$.	Only finding 2 of 4 partial products.	

Multiplying and Dividing Fractions, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
83	Multiplication as Scaling (Part 1)	1	Interpreting multiplication as scaling (resizing).		5.NF.B.5 Interpret multiplication as scaling (resizing), by: 5.NF.B.5.A Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. 5.NF.B.5.B Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
		2	Understanding that $b/b \cdot a = a$. (Targeted Error: Believing $0/a \cdot a = a$.)		
85	Multiplication as Scaling (Part 2)	1	Understanding that, when multiplying a number by an unknown to get an answer that is less than your original number, you must multiply by a number less than 1.		see above
		2	Understanding that multiplying a number by an unknown to get an answer that is greater than your original number, you must multiply by a number greater than 1.	Thinking that multiplying by any number will increase the value of the original number.	
87	Multiplication Word Problems	1	Understanding how to interpret a multi-step word problem.		5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
		2	Understanding how to use conversion factors to solve.	Using the wrong conversion factor.	
89	Area Word Problems with Fractions	1	Interpreting a word problem and determining how to solve for the area of a square.		see above
		2	see above	Adding sides to find perimeter, rather than multiplying to find the area.	

Multiplying and Dividing Fractions, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
91	Division of a Fraction by Whole Number	1	Understanding how to represent dividing fractions using a tape diagram.		5.NF.B.7.A Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.
		2	Understanding how to divide a fraction by a whole number.	Dividing the fraction by $1/a$, rather than $a/1$.	
93	Division of a Whole Number by Fraction	1	Understanding that division does not always lead to a smaller answer.		5.NF.B.7.B Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
		2	Understanding how to match a word problem to a diagram representing a numerical expression.	Misinterpreting the word problem.	
95	Division Word Problems (Part 1)	1	Understanding how to use a diagram and mental math to solve a division word problem.		5.NF.B.7.C Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb. of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?
		2	Understanding how write a division expression based on a word problem.	Switching the dividend and divisor.	
97	Division Word Problems (Part 2)	1	Understanding how to use a number line when dividing fractions.		<i>see above</i>
		2	Understanding that a divided by b/c does not equal $b/(a \text{ divided by } c)$.	Converting a whole number to $1/a$, rather than $a/1$.	

Geometry

Page	Assignment	Set	Target	Targeted Error	CCSS
99	Properties of 2D Figures	1	Understanding that some figures can have more than one name.		5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.; 5.GB.3 Classify two-dimensional figures in a hierarchy based on properties.
		2	Understanding the attributes of quadrilaterals.	Listing incorrect attributes for a rhombus.	

Geometry, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
101	Hierarchy of 2D Figures	1	Understanding that a square is always a rectangle, but a rectangle is not always a square.		5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.
		2	Understanding the attributes of a quadrilateral.	Misidentifying a 5-sided figure as a quadrilateral.	
103	Properties of Volume (Part 1)	1	Understanding that a shape made of "n" unit cubes has the volume of "n" cubic units.		5.MD.C.3.A A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
		2	Understanding how to differentiate shapes with more or less volume.	Identifying a "taller" shape as having more volume without counting the unit cubes.	
105	Properties of Volume (Part 2)	1	Understanding how to find the volume of a net that is folded into a box.	Misinterpreting the definition of volume.	5.MD.C.3.B A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units
		2	Understanding how to determine the volume of a net that is folded into a box, based on the volume of another box.		
107	Counting Unit Cubes (Part 1)	1	Understanding how to find the volume of a rectangular prism by counting unit cubes.	Counting only visible cubes or faces.	5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
		2	Understanding how to find the volume of a rectangular prism by counting and using layers.		
109	Counting Unit Cubes (Part 2)	1	Understanding how to determine the volumes of different shapes.	Failing to recognize that different shapes can have the same volume.	see above
		2	Understanding that a rectangular prism can be partitioned in multiple ways to find volume.		

Geometry, continued

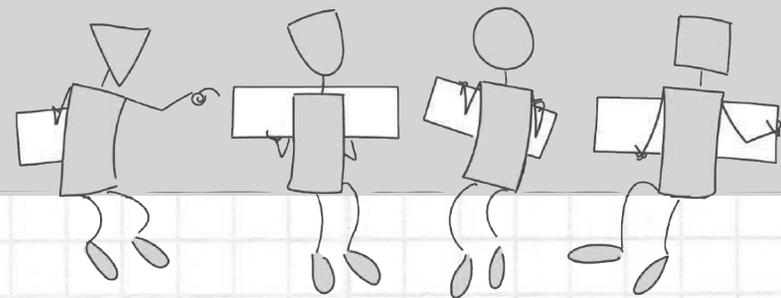
Page	Assignment	Set	Target	Targeted Error	CCSS
111	Packing Unit Cubes (Part 1)	1	Understanding how to find volume by packing a box with unit cubes.		5.MD.C.5.A Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
		2	Understanding how to find the volume of a net that is folded into a box.	Counting all faces of the net to determine volume.	
113	Packing Unit Cubes (Part 2)	1	Understanding how to find the volume of a box.	Multiplying only two of the three dimensions to determine volume.	<i>see above</i>
		2	Determining volume by multiplying the area of the base by the height of the box.		
115	Volume Formula (Part 1)	1	Understanding how to determine the area of the base and the height of a prism.		5.MD.C.5.B Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
		2	Understanding how to apply the formula $V = L \times W \times H$ to find the volume of a prism.	Adding the dimensions of the prism instead of multiplying.	
117	Volume Formula (Part 2)	1	Understanding the relationship between the formula $V = L \times W \times H$ and $V = B \times H$.		<i>see above</i>
		2	Understanding how to determine the volume of a box.	Indicating volume as one dimensional — cm — instead of three dimensional — cm ³ .	
119	Multiple Prisms	1	Understanding that two prisms can have different dimensions but have the same volume.		5.MD.C.5.C Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
		2	Understanding how to find the volume of a shape with unlabeled dimensions.	Ignoring unlabeled dimensions.	
121	Prisms with Unlabeled Dimensions	1	Understanding how to find the volume of a shape with unlabeled dimensions.	Failing to use an equivalent labeled length.	<i>see above</i>
		2	Understanding how to find the volume of a shape with unlabeled dimensions by using labeled dimensions.		

Ordered Pairs

Page	Assignment	Set	Target	Targeted Error	CCSS
123	Graphing on the Coordinate Plane (Part 1)	1	Interpreting points on a coordinate plane.	Reversing the x- and y-coordinates.	5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
		2	Understanding why the ordering of the numbers in an ordered pair changes the position of the point on the coordinate plane.		
125	Graphing on the Coordinate Plane (Part 2)	1	Plotting points on a coordinate plane.	Reversing the x- and y-coordinates.	<i>see above</i>
		2	Understanding that coordinate pair (a, b) is not the same as coordinate pair (b, a).		
127	Graphing on the Coordinate Plane (Part 3)	1	Understanding how to plot a point when 0 is a coordinate.	Plotting the point with 0 as a coordinate on the wrong axis.	<i>see above</i>
		2	Understanding that coordinate pair (a, b) is not the same as coordinate pair (b, a).		
129	Graphing on the Coordinate Plane (Part 4)	1	Understanding how to plot points on a coordinate plane when intervals represent a value less than 1.	Misinterpreting the scale of the coordinate plane.	<i>see above</i>
		2	Determining the scale of a coordinate plane when intervals represent a value less than 1.		
131	Sketching Line Graphs	1	Sketching a line graph based on a word problem.	Not attending to all of the necessary information in a word problem.	5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
		2	Interpreting a word problem and determining an appropriate scale to plot the data.		

Ordered Pairs, continued

Page	Assignment	Set	Target	Targeted Error	CCSS
133	Interpreting Line Graphs	1	Interpreting a line graph.	Associating the biggest decrease with the biggest change without considering change in value.	
		2	Interpreting a line graph to solve problems.	Using the wrong operation to solve a problem.	
135	Generating Number Patterns	1	Determining a rule to describe the relationship between x- and y- coordinates.	Looking at only one ordered pair to determine a rule.	5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
		2	Determining a rule to describe the relationship between x- and y-coordinates.	Switching the x- and y-coordinates when creating a rule.	
137	Graphing Number Patterns	1	Understanding how to apply a rule to determine a second coordinate and graphing the coordinate pairs.		
		2	Understanding how to apply a rule to determine a second coordinate and graph the coordinate pairs.	Ignoring the order of operations when applying the rule.	



Strategic Education Research Partnership
1100 Connecticut Avenue, NW
Suite 1310
Washington, DC 20036
(202) 223-8555
serpinstitute.org